

Board of Education Agenda Item

Item: _____ J. _____

Date: October 23, 2008

Topic: First Review of Proposed Revised *Mathematics Standards of Learning*

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Origin:

_____ Topic presented for information only (no board action required)

X Board review required by
 X State or federal law or regulation
 _____ Board of Education regulation
 _____ Other: _____

X Action requested at this meeting _____ Action requested at future meeting: _____

Previous Review/Action:

_____ No previous board review/action

X Previous review/action
 Date March 19, 2008
 Action Board of Education approved the timeline to proceed with the review process.

Background Information:

The Standards of Learning for mathematics were developed in 1995 and revised in 2001. The *Standards of Quality* require the Board of Education to review the Standards of Learning on a regular schedule. The *Mathematics Standards of Learning* are scheduled for review in 2009. As a result, on March 19, 2008, the Board approved a plan to review these standards during the 2008-2009 academic year. In accordance with the plan, the Department of Education took the following steps to produce a draft of the proposed revised *Mathematics Standards of Learning* for the Board's first review:

- Received online comments from stakeholders, including teachers, parents, and administrators that are summarized in Attachment A;
- Met with a teacher review committee that consisted of recommended individuals solicited from school divisions on August 5, 6, and 7, 2008, to review the public comment and consider recommendations and reports from Achieve, the College Board, ACT, as well as the National Assessment of Educational Progress (NAEP) Frameworks, the Curriculum Focal Points from the National Council of Teachers of Mathematics (NCTM), Principles and Standards for School Mathematics from NCTM, the Singapore Curricula, and the Report of the President's National Mathematics Advisory Panel;

- Solicited a postsecondary review committee comprised of mathematics and mathematics education faculty and met with the review committee on August 20, 2008;
- Solicited a business leaders review committee and sent a summary of the public comment with the current *Mathematics Standards of Learning*, requesting comments; and
- Developed a draft of the proposed revised *Mathematics Standards of Learning*.

Summary of Major Elements:

The attached draft of the proposed revised *Mathematics Standards of Learning* (Attachment B) consists of the following elements:

Introduction

The Standards of Learning for mathematics identify academic content for essential components of the mathematics curriculum at different grade levels for Virginia's public schools. Standards are identified for kindergarten through grade eight and for a core set of high school courses. Throughout a student's mathematics schooling from kindergarten through grade eight, specific content strands or topics are included. These content strands are Number and Number Sense; Computation and Estimation; Measurement; Geometry; Probability and Statistics; and Patterns, Functions, and Algebra. The Standards of Learning for each strand progress in complexity at each grade level and throughout the high school courses.

Goals

The *Mathematics Standards of Learning* address all students' needs today for stronger mathematical knowledge and skills to pursue higher education, to compete in a technologically oriented work force, and to be informed citizens. Students must gain an understanding of fundamental ideas in arithmetic, measurement, geometry, probability, data analysis and statistics, and algebra and functions, and develop proficiency in mathematical skills. In addition, students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. The content of the mathematics standards is intended to support the following five goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Strands/Reporting Categories

The *Mathematics Standards of Learning* for each course are grouped into categories that address related content and skills.

Standards

The *Mathematics Standards of Learning* for Virginia public schools describe the Commonwealth's expectations for student learning and achievement in grades K-12.

Summary of the Proposed Revised *Mathematics Standards of Learning*

The major elements of the attached proposed revised *Mathematics Standards of Learning* include:

- Edits to enhance clarity, specificity, rigor, alignment of skills and content, and a reflection of the current academic research and practice;

- Emphasis on vertical alignment in grades K-7 to prepare students for Algebra I;
- Increased alignment of Algebra I and Algebra II; and
- Increase of focus at each grade level.

Superintendent's Recommendation:

The Superintendent of Public Instruction recommends that the Board of Education accept for first review the proposed revised *Mathematics Standards of Learning*.

Impact on Resources:

This responsibility can be absorbed by the agency's existing resources at this time. If the agency is required to absorb additional responsibilities related to this activity, other services may be impacted.

Timetable for Further Review/Action:

The *Mathematics Standards of Learning* review work plan calls for public hearings, final review, and adoption of the *Mathematics Standards of Learning* by the Board of Education by winter 2009.

Summary of Online Comments**K-12 Mathematics Standards of Learning
March 21, 2008-April 23, 2008**

A total of 957 comments were received electronically for the *Mathematics Standards of Learning* in Kindergarten through Advanced Placement Calculus during the public comment period. No comments were received via U. S. Mail.

The number of comments submitted by grade level or course title is below:

Number of Comments	Grade Level or High School Course Name
87	Kindergarten
81	Grade 1
114	Grade 2
100	Grade 3
122	Grade 4
68	Grade 5
86	Grade 6
65	Grade 7
65	Grade 8
14	Algebra I
9	Geometry
2	Algebra II
4	Advanced Placement Calculus

Nongrade or course specific comments (includes sets of general comments or letters from groups):

Elementary	50
Middle School	21
High School	19

Groups submitting comments were:

- Virginia Council of Mathematics Supervisors
- Alexandria City Public Schools mathematics teachers
- Salem City Public Schools mathematics teachers
- Chesterfield County Public Schools mathematics coordinators and specialists
- Montgomery County Public Schools mathematics teachers
- Stafford County Public Schools mathematics teachers
- Benjamin Franklin Middle School, Franklin County Public Schools, Grade 8 mathematics teachers
- Hanover County Public Schools mathematics specialists and lead mathematics teachers
- Arlington County Public Schools mathematics supervisor and mathematics specialists
- Albemarle County Public Schools mathematics teachers
- Walker Upper Elementary School, Charlottesville City Public Schools, Grade 6 mathematics team

General comments included for the elementary school (K-5) Standards of Learning were:

- Use the Curriculum Focal Points (National Council of Teachers of Mathematics 2007) as a guide for reviewing the Standards of Learning (focus on big ideas; fewer topics in each course in order to provide guidance on the importance of topics);
- Emphasize depth of content rather than breadth of topics; reduce topic redundancy between K-8 and high school courses;
- Increase counting numbers to 200 at Grade 1;
- Develop number relationships between addition and subtraction in contexts and using numerical decomposition;
- Require mathematics facts fluency in specific grades:
 - addition and subtraction to 20 in Grade 3;
 - multiplication and division through 12 x 12 in Grade 4;
- Move metric measurement to the elementary science standards where they are taught in context;
- Use *halves, thirds, fourths, eighths, tenths* rather than the symbols ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{10}$) in the Standards of Learning;
- Add *vertex, vertices, equation*, commutative properties of addition and multiplication, and the additive identity element (0) to Grade 3; associative properties of addition and multiplication and the multiplicative identity element (1) to Grade 4; the distributive property and the multiplicative property of zero to Grade 5; and
- Move percent from the middle grades to Grade 5.

General comments included for the middle school (6-8) Standards of Learning were:

- Use the Curriculum Focal Points (NCTM 2007) as a guide for reviewing the Standards of Learning (focus on big ideas; fewer topics in each course in order to provide guidance on the importance of topics);
- Emphasize depth of content rather than breadth of topics; reduce topic redundancy between K-8 and high school courses;
- Move all the classical constructions to high school Geometry;
- Reduce the number of new topics in Grades 6 and 7;
- Address measurement (U. S. Customary and metric) at all grade levels, K-8;
- Standardize vocabulary K-8;
- Strengthen probability concepts in Grade 8 (theoretical and experimental probability, Law of Large Numbers, counting techniques);
- Address box-and-whisker plots in Grade 8 only.

General comments included for the high school courses were:

- Use the *Algebra, Functions, and Data Analysis Standards of Learning* as a guide for organizing the Standards of Learning in the other high school mathematics courses (focus on big ideas; fewer topics in each course in order to provide guidance on the importance of topics);
- Emphasize depth of content rather than breadth of topics; reduce topic redundancy between K-8 and high school courses;
- Organize strong vertical alignment and articulation between K-8 and Algebra I as well as between Algebra I and Algebra II;
- Provide higher levels of challenge for students;
- Emphasize slope as a rate of change;

- Emphasize a functional approach to algebra content;
- Emphasize practical interpretations and applications of algebra content;
- Move measures of central tendency to middle school and emphasize line of best fit, correlation coefficient, and measures of variability in algebra;
- Remove constructions from Geometry; and
- Delete the *Advanced Placement Calculus Standards of Learning* since the College Board revises the content expectations every two years and publishes a new booklet at that time, but keep the course code.

Proposed Revised Mathematics Standards of Learning

**for
Virginia
Public Schools**

**First Review
October 23, 2008**

Foreword

The Standards of Learning in this publication represent a ~~major~~ significant development in public education in Virginia. These standards focus on the mathematical knowledge and skills all students need for the future, and they have been aligned with national expectations for postsecondary success. The Standards of Learning provide a framework for instructional programs designed to raise the academic achievement of all students in Virginia and are an important part of Virginia's efforts to provide challenging educational programs in the public schools.

The Standards of Learning set reasonable targets and expectations for what teachers need to teach and students need to learn. The standards are not intended to encompass the entire curriculum for a given grade level or course or to prescribe how the content should be taught; the standards are to be incorporated into a broader, locally designed curriculum. Teachers are encouraged to go beyond the standards and select instructional strategies and assessment methods appropriate for their students.

The Standards of Learning are recognized as a model for other states. They were developed through a series of public hearings and the efforts of parents, teachers, representatives from higher education officials, and ~~representatives of~~ business and industry leaders. The standards set clear, concise, and measurable academic expectations for young people. Parents are encouraged to work with their children to help them achieve these academic standards.

A major objective of Virginia's educational agenda is to give the citizens of the eCommonwealth a program of public education that is among the best in the nation and that meets the needs of all young people in the eCommonwealth. These Standards of Learning chart the course for achieving that objective.

Mathematics Standards of Learning

Introduction

The Standards of Learning for mathematics identify academic content for essential components of the mathematics curriculum at different grade levels for Virginia's public schools. Recommendations and reports from Achieve, the College Board, and ACT, as well as the National Assessment of Educational Progress (NAEP) Frameworks, the *Curriculum Focal Points* from the National Council of Teachers of Mathematics (NCTM), *Principles and Standards for School Mathematics* from NCTM, the Singapore Curricula, the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report* from the American Statistical Association, and the *Report of the President's National Mathematics Advisory Panel* were considered in identifying mathematics content necessary for success for all students in postsecondary pursuits.

Standards are identified for kindergarten through grade eight and for a core set of high school courses. Throughout a student's mathematics schooling from kindergarten through grade eight, specific content strands or topics are included. These content strands are Number and Number Sense; Computation and Estimation; Measurement; Geometry; Probability and Statistics; and Patterns, Functions, and Algebra. The Standards of Learning for each strand progress in complexity at each grade level and throughout the high school courses.

The *Mathematics Standards of Learning Curriculum Framework* is a companion document to the *Mathematics Standards of Learning* that amplifies the *Mathematics Standards of Learning* and defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers as they plan their lessons by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the minimum content that all teachers should teach and all students should learn.

The Standards of Learning are not intended to encompass the entire curriculum for a given grade level or course or to prescribe how the content should be taught. Teachers are encouraged to go beyond the standards and to select instructional strategies and assessment methods appropriate for their students.

Goals

Students today require stronger mathematical knowledge and skills to pursue higher education, to compete in a technologically-oriented sophisticated work force, and to be informed citizens. Students must gain an understanding of fundamental ideas in arithmetic, measurement, geometry, probability, data analysis and statistics, and algebra and functions, and develop proficiency in mathematical skills. In addition, students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. Graphing utilities, spreadsheets, calculators, computers, and other forms of electronic information technology are now standard tools for mathematical problem solving in science, engineering, business and industry, government, and practical affairs. Hence, the use of

technology must be an integral part of teaching, ~~and learning, and assessment~~. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. The teaching of computer/technology skills should be the shared responsibility of teachers of all disciplines.

The content of the mathematics standards is intended to support the following five goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-life data and situations within and outside mathematics and then apply appropriate strategies to find an acceptable solution. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problem types. A major goal of the mathematics program is to help students become competent mathematical problem solvers.

Mathematical Communication

Students will use the language of mathematics, including specialized vocabulary and symbols, to express mathematical ideas precisely. Representing, discussing, reading, writing, and listening to mathematics will help students to clarify their thinking and deepen their understanding of the mathematics being studied.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will learn to apply proportional and spatial reasoning and to reason from a variety of representations such as graphs, tables, and charts.

Mathematical Connections

Students will relate concepts and procedures from different topics in mathematics to one another and see mathematics as an integrated field of study. Through the application of content and process skills, students will make connections between different areas of mathematics and between mathematics and other disciplines, especially science. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships with a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should move easily among different representations—graphical, numerical, algebraic, verbal, and physical—and recognize that representation is both a process and a product.

Kindergarten

The kindergarten standards place emphasis on developing the concept of number by counting; combining, sorting, and comparing sets of objects; recognizing and describing simple repeating patterns; and recognizing shapes and sizes of figures and objects. Students will investigate nonstandard and standard measurement, collect data, and create graphs. The idea of fractions is introduced.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense (Focus: Whole Number Concepts)

- K.1 The student, given two sets containing ~~10~~20 or fewer concrete objects, will identify and describe one set as having more, fewer, or the same number of members as the other set, using the concept of one-to-one correspondence.
- K.2 The student, given a set containing ~~10~~20 or fewer concrete items, will
 - a) tell how many are in the set by counting the number of items orally;
 - b) select the corresponding numeral from a given set; and
 - c) write the numeral to tell how many are in the set.
- K.3 The student, given an ordered set of ~~three~~ ten objects and/or pictures, will indicate the ordinal position of each item, first through ~~third~~ tenth, and the ordered position of each item from left-to-right, right-to-left, top-to-bottom, and/or bottom-to-top.
- ~~K.4 The student will investigate and recognize patterns from counting by fives and tens to 30, using concrete objects and a calculator. [Moved to new SOL K.4 c]~~

- ~~K.5~~ K.4 The student will ~~count~~
 a) ~~count~~ forward to ~~30~~ 100 and backward from ~~40~~ 30;
 b) ~~identify one more than and one less than a number; and~~
 c) ~~count by fives and tens to 30-100 using concrete objects and a calculator.~~ [Move to Curriculum Framework]

- K.5 The student will identify the part of a set and/or region that represents a fraction for halves and fourths.

Computation and Estimation **(Focus: Whole Number Operations)**

- K.6 The student will model adding and subtracting whole numbers, ~~using up to 10 concrete items~~ using whole numbers up to ten.

Measurement **(Focus: Instruments and Attributes)**

- K.7 The student will recognize a penny, nickel, dime, and quarter and will determine the value of a collection of pennies and/or nickels whose total value is 10 cents or less.
- K.8 The student will identify the instruments used to measure length (ruler), weight (scale), time (clock: digital and analog; calendar: day, month, and season), and temperature (thermometer).
- K.9 The student will tell time to the hour, using an analog ~~or~~ and digital clock.
- K.10 The student will compare two objects or events, using direct comparisons or nonstandard units of measure, according to one or more of the following attributes: length (shorter, longer), height (taller, shorter), weight (heavier, lighter), temperature (hotter, colder). Examples of nonstandard units include foot length, hand span, new pencil, paper clip, block.

Geometry **(Focus: Identify Plane Shapes)**

- K.11 The student will
 a) ~~identify, and describe, and draw~~ two-dimensional (plane) geometric figures (circle, triangle, square, and rectangle); and
 b) compare the size (larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle).
- K.12 The student will describe the location of one object relative to another (above, below, next to) and identify representations of plane geometric figures (circle, triangle, square, and rectangle) regardless of their position and orientation in space.

~~K.13~~ The student will compare the size (larger, smaller) and shape of plane geometric figures (circle, triangle, square, and rectangle). [Moved to new SOL K.11 b]

Probability and Statistics **(Focus: Data Collection and Display)**

~~K.14~~ K.13 The student will gather data relating to familiar experiences by counting and tallying.

~~K.15~~ K.14 The student will display gathered data in objects and information, using object graphs, pictorial picture graphs, and tables, and will answer questions related to the data.

~~K.16~~ K.15 The student will investigate and describe the results of dropping a two-colored counter or using a multicolored spinner.

Patterns, Functions, and Algebra **(Focus: Attributes and Patterning)**

~~K.17~~ K.16 The student will sort and classify objects according to similar attributes (size, shape, and color). [Move to Curriculum Framework]

~~K.18~~ K.17 The student will identify, describe, and extend a repeating and growing relationship patterns found in common objects, sounds, and movements. [Move to Curriculum Framework]

Grade One

The first-grade standards place emphasis on counting, sorting, and comparing sets of up to 100 objects; recognizing and describing simple repeating and growing patterns; and ~~drawing, describing and~~ sorting ~~two-dimensional plane geometric~~ figures. Students' understanding of number is expanded through learning and applying the basic addition facts through the ~~fives~~ nines table and the corresponding subtraction facts; using nonstandard and standard units to measure; and organizing and interpreting data. ~~The idea of fractions is introduced.~~ [Moved to Kindergarten.] Fractional concepts are expanded.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense (Focus: Place Value and Fraction Concepts)

- 1.1 The student will
 - a) ~~count objects in a given set containing between 1 and~~ from 0 to 100 objects and write the corresponding numeral; and
 - b) group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.
- ~~1.2 The student will group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.~~ [Moved to new SOL 1.1 b]
- ~~1.3~~ 1.2 The student will count forward by ones, twos, fives, and tens to 100, ~~by twos to 20~~ and backward by ones from ~~20~~ 30.
- ~~1.4 The student will recognize and write numerals 0 through 100.~~ [Moved to new SOL 1.1 a]

- 1.5 ~~The student will identify the ordinal positions first through tenth, using an ordered set of objects. [Moved to new SOL K.3]~~
- 1.6 1.3 ~~The student will identify and represent the concepts of one half and one fourth, using appropriate materials or a drawing. the part of a set and/or region that represents a fraction for halves, thirds, and fourths and write the fraction.~~

Computation and Estimation (Focus: Whole Number Operations)

- 1.7 1.4 The student, given a familiar problem situation involving magnitude, will
 a) select a reasonable magnitude from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, and 500); and
 b) explain the reasonableness of his/her choice.
- 1.8 1.5 The student will recall basic addition facts, i.e., sums to ~~40~~ 18 or less and the corresponding subtraction facts.
- 1.9 1.6 The student will create and solve story and picture problems involving one-step solutions, using addition facts with sums to 18 or less and the corresponding subtraction facts.

Measurement (Focus: Time and Measurement)

- 1.10 1.7 The student will
 a) identify the number of pennies equivalent to a nickel, a dime, and a quarter; and
 b) determine the value of a collection of pennies, nickels, and dimes whose total value is 100 cents or less.
- 1.11 1.8 The student will tell time to the half-hour, using an analog ~~or~~ and digital clock.
- 1.12 1.9 The student will use nonstandard units to measure length, ~~and~~ weight/mass and volume.
- 1.13 1.10 The student will compare
 a) the volumes of two given containers ~~by using concrete materials (e.g., jelly beans, sand, water, rice)~~ [Move to Curriculum Framework] ~~;~~ and
 b) the weights of two objects, using a balance scale.
- 1.14 ~~The student will compare the weights of two objects, using a balance scale. [Moved to new SOL 1.10 b]~~
- 1.11 The student will use calendar language appropriately (e.g., *months, today, yesterday, next week, last week*).

Geometry

(Focus: Geometric Property Development)

- 1.15 ~~The student will describe the proximity of objects in space (*near, far, close by, below, above, up, down, beside, and next to*).~~ [Move to Curriculum Framework]
- 1.16 ~~1.12~~ The student will ~~draw~~, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, corners, and ~~square corners~~ [Move to Curriculum Framework] right angles.
- 1.17 ~~1.13~~ The student will ~~identify~~ construct, model, and describe objects in ~~his/her the~~ environment using shapes and spatial reasoning ~~that depict plane geometric figures (triangle, rectangle, square, and circle).~~ [Move to Curriculum Framework]

Probability and Statistics

(Focus: Data Collection and Interpretation)

- 1.18 ~~1.14~~ The student will investigate, identify, and describe various forms of data collection in his/her world (e.g., recording daily temperature, lunch count, attendance, and favorite ice cream), using tables, picture graphs, and object graphs.
- 1.19 ~~1.15~~ The student will interpret information displayed in a picture or object graph, using the vocabulary *more, less, fewer, greater than, less than, and equal to*.

Patterns, Functions, and Algebra

(Focus: Patterning and Equivalence)

- 1.20 ~~1.16~~ The student will sort and classify concrete objects according to one or more attributes, including color, size, shape, and thickness.
- 1.21 ~~1.17~~ The student will recognize, describe, extend, and create a wide variety of patterns including ~~rhythmic, color, shape, and numerical~~. ~~Patterns will include both~~ [Move to Curriculum Framework] growing and repeating patterns. ~~Concrete materials and calculators will be used by students~~ [Move to Curriculum Framework]
- 1.18 The student will recognize the equal sign as a representation of equivalency.

Grade Two

The second-grade standards extend the study of number and spatial sense to include three-digit numbers and solid geometric (~~three-dimensional~~) figures. Students will continue to learn, use, and gain proficiency in the basic addition facts through the nines table and the corresponding subtraction facts. Students will begin to use standard U.S. Customary and metric units of measurement; predict, using simple probability; and create and interpret picture and bar graphs. Students will work with a variety of patterns and will develop knowledge of equality by identifying missing numbers in addition and subtraction facts.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense

(Focus: Place Value, Number Patterns, and Fraction Concepts)

- 2.1 The student will
- a) read, write, and identify the place value of each digit in a three-digit numeral, using numeration models; ~~and~~
 - b) round two-digit numbers to the nearest ten; and
 - c) compare two whole numbers between 0 and 999, using symbols ($>$, $<$, or $=$) and words (*greater than*, *less than*, or *equal to*).
- 2.2 ~~The student will compare two whole numbers between 0 and 999, using symbols ($>$, $<$, or $=$) and words (*greater than*, *less than*, or *equal to*).~~ [Moved to new SOL 2.1 c]
- 2.3 2.2 The student will
- a) identify the ordinal positions first through twentieth, using an ordered set of objects; and-
 - b) write the ordinal numbers.

- 2.4 2.3 The student will
- identify the part of a set and/or region that represents ~~a~~ fractions for ~~one-half, one-third, one-fourth, one-eighth, and one-tenth~~ halves, thirds, fourths, sixths, eighths, and tenths;
 - write the ~~corresponding~~ fraction; and-
 - compare the unit fractions for halves, thirds, fourths, sixths, eighths, and tenths.
- 2.5 2.4 The student will
- count forward by twos, fives, and tens to 100, starting at various multiples of 2, 5, or 10; ~~using mental mathematics, paper and pencil, hundred chart, calculators, and/or concrete objects, as appropriate~~ [Move to Curriculum Framework] ;
 - count backward by tens from 100;
 - group objects by threes and fours; and
 - recognize even and odd numbers, ~~using objects.~~ [Move to Curriculum Framework]

Computation and Estimation

(Focus: Number Relationships and Operations)

- 2.6 2.5 The student will recall basic addition facts, i.e., sums to ~~18~~ 20 or less and the corresponding subtraction facts.
- 2.7 2.6 The student, given two whole numbers whose sum is 99 or less, will
- estimate the sum; and
 - find the sum, using various methods of calculation, ~~(mental computation, concrete materials, and paper and pencil).~~ [Move to Curriculum Framework]
- 2.8 2.7 The student, given two whole numbers, each of which is 99 or less, will
- estimate the difference; and
 - find the difference, using various methods of calculation, ~~(mental computation, concrete materials, and paper and pencil.)~~ [Move to Curriculum Framework]
- 2.9 2.8 The student will create and solve ~~one- or two-step~~ addition and subtraction problems using data from simple tables, picture graphs, and bar graphs, ~~and practical situations.~~ [Move to Curriculum Framework]
- 2.10 2.9 The student, ~~given a simple addition or subtraction fact,~~ will recognize and describe the related facts which represent and describe the inverse relationship between addition and subtraction (e.g., $3 + \underline{\quad} = 7$, $\underline{\quad} + 3 = 7$; $7 - 3 = \underline{\quad}$, and $7 - \underline{\quad} = 3$).

Measurement

(Focus: Money, Time, and Measurement)

- ~~2.11~~ 2.10 The student will
- count and compare a collection of pennies, nickels, dimes, and quarters whose total value is \$2.00 or less; and
 - ~~identify the correct usage of~~ correctly use the cent symbol (¢), dollar symbol (\$), and decimal point (.).
- ~~2.12~~ 2.11 The student will estimate and ~~then use a ruler to make linear measurements to~~ measure
- ~~to the nearest centimeter and inch, including measuring the distance around a polygon in order to determine perimeter.~~ [Moved to new SOL 3.10 a]
 - weight/mass of objects using a scale in pounds, ounces/kilograms, grams; and
 - liquid volume in (cups, pints, quarts, gallons, and liters), using the concepts of more, less, and equivalent.
- ~~2.13~~ The student, given grid paper, will estimate and then count the number of square units needed to cover a given surface in order to determine area. [Moved to new SOL 5.9 a]
- ~~2.14~~ The student will estimate and then count the number of cubes in a rectangular box in order to determine volume. [Moved to new SOL 5.9 a]
- ~~2.15~~ The student will estimate and then determine weight/mass of familiar objects in pounds and/or kilograms. [Moved to new SOL 2.11 b]
- ~~2.16~~ 2.12 The student will tell and write time to the ~~quarter hour~~ nearest five minutes, using analog and digital clocks.
- ~~2.17~~ The student will use actual measuring devices to compare metric and U.S. Customary units (cups, pints, quarts, gallons, and liters) for measuring liquid volume, using the concepts of *more, less, and equivalent*. [Moved to new SOL 2.11 c]
- ~~2.18~~ 2.13 The student will
- ~~use calendar language appropriately (e.g., months, today, yesterday, next week, last week);~~ [Moved to new SOL 1.11]
 - ~~b) a) determine past and future days of the week; and~~
 - ~~e) b) identify specific days and dates on a given calendar.~~
- ~~2.19~~ 2.14 The student will read the temperature on a Celsius and/or Fahrenheit thermometer to the nearest 10 degrees.

Geometry

(Focus: Symmetry and Plane and Solid Figures)

- ~~2.20~~ The student will identify, describe, and sort three-dimensional (solid) concrete figures; [Moved to new SOL 3.14] ~~including a cube, rectangular solid (prism), square pyramid, sphere, cylinder, and cone, according to the number and shape of the solid's faces, edges, and corners. [Move to Curriculum Framework in support of new SOL 3.14]~~
- ~~2.21~~ 2.15 The student will
- a) draw a line of symmetry in a figure; and
 - b) identify and create figures symmetric along a line, with at least one line of symmetry using various concrete materials. [Move to Curriculum Framework]
- ~~2.22~~ 2.16 The student will identify, describe, compare, and contrast plane and solid geometric shapes (circle/sphere, square/cube, and rectangle /rectangular solid prism).

Probability and Statistics

(Focus: Applications of Data)

- ~~2.23~~ 2.17 The student will use data from experiments to read, construct, and interpret a simple picture graphs and bar graphs.
- ~~2.24~~ 2.18 The student will ~~record~~ use data from experiments, using spinners and colored tiles/cubes, and use the data to predict outcomes which of two events is more likely to occur [Move to Curriculum Framework] if the experiment is repeated.
- 2.19 The student will analyze data displayed in a picture graph and bar graph.

Patterns, Functions, and Algebra

(Focus: Patterning and Numerical Sentences)

- ~~2.25~~ 2.20 The student will identify, create, and extend a wide variety of patterns, ~~using numbers, concrete objects and pictures.~~ [Move to Curriculum Framework]
- ~~2.26~~ 2.21 The student will solve problems by completing a numerical sentence involving the basic facts for addition and subtraction. ~~Examples include: $3 + \underline{\quad} = 7$, or $9 - \underline{\quad} = 4$, or $\underline{\quad} - 2 = 7$.~~ [Move to Curriculum Framework] Students will create story problems, using the numerical sentences.

Grade Three

The third-grade standards place emphasis on learning multiplication and division facts through the nines table. Students will be fluent in the basic addition facts through the nines table and the corresponding subtraction facts. Concrete materials and two-dimensional representations will be used to introduce addition and subtraction with fractions and decimals and the concept of probability as chance. Students will use standard units (U.S. Customary and metric) for temperature, length, liquid volume, and weight and identify relevant properties of shapes, points, line segments, rays, and angles, and lines. Students will investigate and describe the identity and commutative properties for addition and multiplication.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense (Focus: Place Value and Fractions)

- 3.1 The student will
- a) read and write six-digit numerals and identify the place value for each digit;
 - b) round a whole number, 9,999 or less, to the nearest ten, hundred, and thousand;
and
 - c) compare two whole numbers between 0 and 9,999, using symbols ($>$, $<$, or $=$) and words (*greater than*, *less than*, or *equal to*).
- 3.2 ~~The student will round a whole number, 9,999 or less, to the nearest ten, hundred, and thousand.~~ [Moved to new SOL 3.1 b]
- 3.3 ~~The student will compare two whole numbers between 0 and 9,999, using symbols ($>$, $<$, or $=$) and words (*greater than*, *less than*, or *equal to*).~~ [Moved to new SOL 3.1 c]

- 3.4 3.2 The student will recognize and use the inverse relationships between addition/subtraction and multiplication/division to complete basic fact sentences. Students will use these relationships to solve problems such as $5 + 3 = 8$ and $8 - 3 = \underline{\quad}$. [Move to Curriculum Framework]
- 3.5 3.3 The student will
- a) ~~divide regions and sets to represent a fraction; and~~ [Move to Curriculum Framework]
 - b) ~~name and write the fractions represented by a given model (area/region, length/measurement, and set). Fractions (including mixed numbers) will include halves, thirds, fourths, eighths, and tenths.~~ [Moved to new SOL 3.3 a]
 - a) name and write fractions (including mixed numbers) represented by a model;
 - b) model fractions (including mixed numbers) and write the fractions' names; and
 - c) compare the numerical value of two fractions having like and unlike denominators, using words and symbols for $<$, $>$, and $=$, using concrete or pictorial models involving areas/regions and lengths/measurements. [Move to Curriculum Framework]
- 3.6 ~~The student will compare the numerical value of two fractions having like and unlike denominators, using concrete or pictorial models involving areas/regions, lengths/measurements, and sets.~~ [Moved to new SOL 3.3 c]
- 3.7 ~~The student will read and write decimals expressed as tenths and hundredths, using concrete materials and models.~~ [Moved to new SOL 4.3 a]

Computation and Estimation

(Focus: Computation and Fraction Operations)

- 3.8 3.4 The student will solve single and multistep problems involving the sum or difference of two whole numbers, each 9,999 or less, with or without regrouping ~~using various computational methods, including calculators, paper and pencil, mental computation, and estimation.~~ [Move to Curriculum Framework]
- 3.9 3.5 The student will
- a) ~~recall the multiplication and division facts through the ~~nines~~ twelves table; and~~
 - b) find the related quotients.
- 3.10 3.6 The student will represent multiplication and division, using area ~~and~~ set, and number line models; and create and solve problems that involve multiplication of two whole numbers, one factor 99 or less and the second factor 5 or less.

- ~~3.11~~ 3.7 The student will add and subtract with proper fractions having like denominators of ~~10~~ 12 or less, ~~using concrete materials and pictorial models representing areas/regions, lengths/measurements, and sets.~~ [Move to Curriculum Framework]
- ~~3.12~~ The student will add and subtract with decimals expressed as tenths, ~~using concrete materials, pictorial representations, and paper and pencil.~~ [Moved to new SOL 4.5 c]

Measurement

(Focus: U.S. Customary and Metric Units, Area and Perimeter, Time)

- ~~3.13~~ 3.8 The student will determine by counting the value of a collection of bills and coins whose total value is \$5.00 or less, compare the value of the coins or bills, and make change.
- ~~3.14~~ 3.9 The student will estimate and ~~then use actual measuring devices with metric and U.S. Customary~~ U.S. Customary and metric units to measure:
- a) length — including part of an inch ($\frac{1}{2}$), inches, feet, yards, centimeters, and meters;
 - b) liquid volume — cups, pints, quarts, gallons, and liters; ~~and~~
 - c) weight/mass — ounces, pounds, grams, and kilograms; ~~and~~
 - d) area and perimeter.
- 3.10 The student will
- a) measure the distance around a polygon in order to determine perimeter; and
 - b) count the number of square units needed to cover a given surface in order to determine area.
- ~~3.15~~ 3.11 The student will
- a) tell time to ~~the nearest five-minute interval and to~~ [Moved to new SOL 2.12] the nearest minute, using analog and digital clocks; ~~and~~
 - b) determine elapsed time in one-hour increments over a 12-hour period.
- ~~3.16~~ 3.12 The student will identify equivalent periods of time, including relationships among days, months, and years, as well as minutes and hours.
- ~~3.17~~ 3.13 The student will read temperature to the nearest degree from a Celsius thermometer and a Fahrenheit thermometer. Real thermometers and physical models of thermometers will be used.

Geometry

(Focus: Properties and Congruence)

- ~~3.18~~ 3.14 The student will identify, describe, compare, and contrast ~~analyze~~ characteristics of ~~two-dimensional~~ (plane) and ~~three-dimensional~~ (solid) geometric figures (circle, square, rectangle, triangle, cube, rectangular ~~solid~~ [prism], square pyramid, sphere, cone, and cylinder).
- ~~3.19~~ 3.15 The student will identify and draw representations of points, line segments, rays, angles, and lines [Move to Curriculum Framework].
- ~~3.20~~ 3.16 The student, ~~given appropriate drawings or models~~, [Move to Curriculum Framework] will identify and describe congruent and symmetrical, noncongruent, ~~two-dimensional~~ (plane) (two-dimensional) figures, ~~using tracing procedures~~. [Move to Curriculum Framework]

Probability and Statistics

(Focus: Applications of Data and Chance)

- ~~3.21~~ 3.17 The student, ~~given grid paper~~, will
- collect and organize data on a ~~given~~ topic of his/her choice, using observations, measurements, surveys, or experiments; ~~and~~
 - construct a line plot, a picture graph, or a bar graph to represent ~~the results~~; the data ~~Each graph will include an appropriate title and key.~~ [Move to Curriculum Framework]; and
 - read and interpret the data represented in line plots, bar graphs, and picture graphs and write a sentence analyzing the data.
- ~~3.22~~ The student will read and interpret data represented in line plots, bar graphs, and picture graphs and write a sentence analyzing the data. [Moved to new SOL 3.17 c]
- ~~3.23~~ 3.18 The student will investigate and describe the concept of probability as chance and list possible results of a given situation.

Patterns, Functions, and Algebra

(Focus: Patterns and Property Concepts)

- ~~3.24~~ 3.19 The student will recognize and describe a variety of patterns formed using ~~concrete objects~~, numbers, tables, and pictures, and extend the pattern, using the same or different forms.

~~3.25~~ 3.20 The student will

- a) ~~investigate and create patterns involving numbers, [Move to Curriculum Framework] operations (addition and multiplication), and relations that model the identity and the commutative properties for addition and multiplication; and~~
- b) identify examples of the identity and commutative properties for addition and multiplication; and [This addition is to clarify SOL]
- ~~b)c)~~ demonstrate an understanding of equality by recognizing that the equals sign (=) in an equation links equivalent quantities, ~~such as $4 + 3 = 2 + 6$.~~ [Move to Curriculum Framework]

Grade Four

The fourth-grade standards place emphasis on multiplication and division with whole numbers and solving problems involving addition and subtraction of fractions and decimals. ~~Students will continue to learn and use the basic multiplication facts.~~ Students will be fluent in the basic multiplication facts through the twelves table and the corresponding division facts as they become proficient in multiplying larger numbers. Students also will refine their estimation skills for computations and measurements ~~and investigate relationships between and among simple two-dimensional (plane) figures and three-dimensional (solid) figures.~~ Students will identify and ~~draw~~ describe representations of points, lines, line segments, ~~and rays, and angles, including endpoints and vertices.~~ ~~Students will graph points in the first quadrant in the coordinate plane and extend and duplicate patterns.~~ Concrete materials and two-dimensional representations will be used to solve problems involving perimeter, patterns, probability, and equivalence of fractions and decimals. Students will recognize images of figures resulting from a geometric transformations, such as reflection (flip), translation (slide), and rotation (turn). Students will investigate and describe the associative property for addition and multiplication.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense (Focus: Place Value, Fractions, and Decimals)

- 4.1 The student will
- identify (orally and in writing) the place value for each digit in a whole number expressed through millions;
 - compare two whole numbers expressed through millions, using symbols ($>$, $<$, or $=$); and
 - round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

- 4.2 The student will
- a) ~~identify, model, and compare and order rational numbers~~ fractions and mixed numbers; ~~using concrete objects and pictures~~ [Move to Curriculum Framework]
 - b) represent equivalent fractions; and
 - e) ~~relate fractions to decimals, using concrete objects.~~ [Moved to new SOL 4.3 d]
 - c) identify the fraction that represents division.
- 4.3 ~~The student will compare the numerical value of fractions~~ [Moved to new SOL 4.2 a] ~~(with like and unlike denominators) having denominators of 12 or less, using concrete materials.~~ [Move to Curriculum Framework]
- 4.4 4.3 The student will
- a) read, write, represent, and identify decimals expressed ~~as tenths and hundredths through thousandths~~;
 - b) round decimals to the nearest whole number, tenth, and hundredth; ~~and~~
 - c) compare ~~the value of two~~ and order decimals, ~~using symbols ($<$, $>$, or $=$), concrete materials, drawings, and calculators.~~ [Move to Curriculum Framework] ; and
 - d) given a model, write the decimal and fraction equivalents.

Computation and Estimation

(Focus: Whole Number, Fraction, and Decimal Operations, and Estimation)

- 4.5 4.4 The student will ~~estimate whole number sums and differences and describe the method of estimation. Students will refine estimates, using terms such as *closer to*, *between*, and *a little more than*.~~ [Move to Curriculum Framework]
- a) estimate sums, differences, products, and quotients for whole numbers;
 - b) add, subtract, and multiply whole numbers;
 - c) divide whole numbers, finding quotients with and without remainders; and
 - d) solve single and multistep addition, subtraction, and multiplication problems with whole numbers.
- 4.6 ~~The student will add and subtract whole numbers written in vertical and horizontal form, choosing appropriately between paper and pencil methods and calculators.~~ [Move to Curriculum Framework]
- 4.7 ~~The student will find the product of two whole numbers when one factor has two digits or fewer and the other factor has three digits or fewer,~~ [Moved to new SOL 4.4 a] ~~using estimation and paper and pencil. For larger products (a two-digit numeral times a three-digit numeral), estimation and calculators will be used.~~ [Move to Curriculum Framework]
- 4.8 ~~The student will estimate and find the quotient of two whole numbers~~ [Moved to new SOL 4.4 c], ~~given a one-digit divisor.~~ [Move to Curriculum Framework]

- 4.9 ~~4.5~~ The student will
- a) ~~add and subtract with fractions having like and unlike denominators of 12~~
[Moved to new SOL 4.5 b] ~~or less, using concrete materials, pictorial representations, and paper and pencil; simplify fractions;~~
 - b) ~~add and subtract with decimals~~ [Moved to new SOL 4.5 c] ~~through thousandths, using concrete materials, pictorial representations, and paper and pencil; add and subtract with fractions having like and unlike denominators;~~
 - c) ~~solve problems involving addition and subtraction with fractions~~ [Moved to new SOL 4.5 d] ~~having like and unlike denominators of 12 or less~~ [Move to Curriculum Framework] ~~and with decimals~~ [Moved to new SOL 4.5 d] ~~expressed through thousandths, using various computational methods, including calculators, paper and pencil, mental computation, and estimation~~ [Move to Curriculum Framework] add and subtract with decimals; and
 - d) solve single-step practical problems involving addition and subtraction with fractions and with decimals.

Measurement

(Focus: Equivalence between U.S. Customary and Metric Units)

- 4.10 ~~4.6~~ The student will
- a) ~~estimate and measure weight/mass, using actual measuring devices;~~ [Move to Curriculum Framework] ~~and describe the results in U.S. Customary/metric units as appropriate, including ounces, pounds, tons, grams, and kilograms~~ [Move to Curriculum Framework]; and
 - b) ~~identify equivalent measurements between units within the U.S. Customary system (ounces, ~~and~~ pounds, and tons, and between units within the metric system (grams and kilograms).; ~~and~~~~
 - c) ~~estimate the conversion of ounces and grams and pounds and kilograms, using approximate comparisons (1 ounce is about 28 grams, or 1 gram is about the weight of a paper clip; 1 kilogram is a little more than 2 pounds).*~~
[Moved to new SOL 6.9]

**The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U.S. Customary and metric units.*

~~4.11~~ 4.7 The student will

- a) estimate and measure length, ~~using actual measuring devices~~, and determine the result in both metric and U.S. Customary units including part of an inch ($\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$), inches, feet, yards, miles, millimeters, centimeters, and meters [Move to Curriculum Framework]; and
- b) identify equivalent measurements between units within the U.S. Customary system (inches and feet; feet and yards; inches and yards; yards and miles) and between units within the metric system (millimeters and centimeters; centimeters and meters; and millimeters and meters); ~~and~~
- c) ~~estimate the conversion of inches and centimeters, yards and meters, and miles and kilometers, using approximate comparisons (1 inch is about 2.5 centimeters, 1 meter is a little longer than 1 yard, 1 mile is slightly farther than 1.5 kilometers, or 1 kilometer is slightly farther than half a mile).~~*
[Moved to new SOL 6.9]

** The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U.S. Customary and metric units.*

~~4.12~~ 4.8 The student will

- a) estimate and measure liquid volume, ~~using actual measuring devices~~ [Move to Curriculum Framework] and ~~using metric and~~ describe the results in U.S. Customary; and
- b) identify equivalent measurements between units within the U.S. Customary system (cups, pints, quarts, and gallons); ~~and between units within the metric system (milliliters and liters); and~~
- c) ~~estimate the conversion of quarts and liters, using approximate comparisons (1 quart is a little less than 1 liter, 1 liter is a little more than 1 quart).~~* [Moved to new SOL 6.9]

** The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U. S. Customary and metric units.*

~~4.13~~ ~~The student will~~

- a) ~~identify and describe situations representing the use of perimeter and area;~~
[Moved to new SOL 5.9 a] and
- b) ~~use measuring devices to find perimeter in both standard and nonstandard units of measure.~~[Moved to new SOL 5.9 a]

4.9 The student will determine elapsed time in hours and minutes within a 12-hour period.

Geometry

(Focus: Representations and Polygons)

- 4.14 4.10 The student will ~~investigate and describe the relationships between and among points, lines, line segments, and rays.~~
- a) identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices;
 - b) identify representations of lines that illustrate intersection, parallelism, and perpendicularity; and
 - c) describe the path of shortest distance between two points on a plane surface.
- 4.15 The student will
- a) ~~identify and draw representations of points, lines, line segments, rays, and angles, using a straightedge or ruler; and [Moved to new SOL 4.10 a]~~
 - b) ~~describe the path of shortest distance between two points on a flat surface. [Moved to new SOL 4.10 c]~~
- 4.16 The student will ~~identify and draw representations of lines that illustrate intersection, parallelism, and perpendicularity. [Moved to new SOL 4.10 b]~~
- 4.17 4.11 The student will
- a) ~~analyze and compare the properties of two-dimensional (plane) geometric figures (circle, square, rectangle, triangle, parallelogram, and rhombus) and three-dimensional (solid) geometric figures (sphere, cube, and rectangular solid [prism]); [Moved to new SOL 3.14]~~
 - b) ~~identify congruent and noncongruent shapes; and [Moved to new SOL 3.16]~~
 - a) investigate congruence of plane figures after geometric transformations such as reflection (flip), translation (slide) and rotation (turn), using mirrors, paper folding, and tracing.
 - b) recognize the images of figures resulting from geometric transformations such as translation (slide), reflection (flip), or rotation (turn).
- 4.18 The student will ~~identify the ordered pair for a point and locate the point for an ordered pair in the first quadrant of a coordinate plane. [Moved to new SOL 6.11]~~
- 4.12 The student will
- a) define polygon; and
 - b) identify polygons with 10 or fewer sides.

Probability and Statistics

(Focus: Outcomes and Congruent and Noncongruent Regions)

- 4.19 ~~4.13~~ The student will
- predict the likelihood of an outcomes of a simple event, [Moved to new SOL 2.18] ~~using the terms *certain, likely, unlikely, impossible*~~ [Move to Curriculum Framework in support of new SOL 2.18]
 - determine ~~the probability~~ probabilities of a given simple event, using concrete materials, geometric representations with congruent and noncongruent regions; and
 - represent probability as a number between 0 and 1.
- 4.20 ~~4.14~~ The student will collect, organize, and display data in line ~~and bar~~ graphs ~~with scale increments of one or greater than one~~ [Move to Curriculum Framework] and use the display to interpret the results, draw conclusions, and make predictions.

Patterns, Functions, and Algebra

(Focus: Geometric Patterns, Equality, Properties)

- 4.21 ~~4.15~~ The student will recognize, create, and extend numerical and geometric patterns, ~~using concrete materials, number lines, symbols, tables, and words.~~ [Move to Curriculum Framework]
- 4.22 ~~4.16~~ The student will
- recognize and demonstrate the meaning of equality in an equation, ~~using symbols representing numbers, operations, and relations [e.g., $3 + 5 = 5 + 3$ and $15 + (35 + 16) = (15 + 35) + 16$]~~ [Move to Curriculum Framework]; and
 - investigate and describe the associative property for addition and multiplication.

Grade Five

The fifth-grade standards place emphasis on developing proficiency in using whole numbers including prime and composite numbers, finding common multiples and factors, identifying even numbers with and without remainders and solving problems using order of operations for positive whole numbers. Students will develop proficiency in the use of fractions, and decimals to solve problems. Students will collect, display, and analyze data in a variety of ways and solve probability problems, using a sample space or tree diagram. Students also will solve problems involving volume, area, and perimeter. Students will be introduced to variable expressions and open sentences, and model one-step linear equations in one variable using addition and subtraction. Students will investigate and recognize the distributive property. All of these skills will assist to develop the algebraic concepts needed for success in pre-algebra at the middle level.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as calculators and computers. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense

(Focus: Factors and Multiples, Fractions, Decimals)

- 5.1 The student, given a decimal through thousandths, will round to the nearest whole number, tenth, or hundredth.
- a) ~~read, write, and identify the place values of decimals through thousandths;~~
 - b) ~~round decimal numbers to the nearest tenth or hundredth; and~~
 - c) ~~compare the values of two decimals through thousandths, using the symbols $>$, $<$, or $=$.~~ [Moved to new SOL 4.3]

- 5.2 The student will
- recognize and name ~~commonly used~~ fractions (~~halves, fourths, fifths, eighths, and tenths~~) [Move to Curriculum Framework] in their equivalent decimal form and vice versa; and
 - compare and order a given set of fractions and decimals from least to greatest. Fractions will include like and unlike denominators limited to 12 or less, and mixed numbers. [Move to Curriculum Framework]

- 5.3 The student will
- find common multiples and factors, including least common multiple and greatest common factor;
 - identify and describe prime and composite numbers; and
 - identify even numbers as having no remainder when divided by two and odd numbers as having a remainder of one when divided by two.

Computation and Estimation

(Focus: Computation Operations and Estimations)

- 5.3 5.4 The student will
- create and solve single and multistep practical problems involving addition, subtraction, multiplication, and division with and without remainders of whole numbers, ~~using paper and pencil, estimation, mental computation, and calculators.~~[Move to Curriculum Framework]; and
 - find the sum, difference, product, and quotient of two numbers expressed as decimals through thousandths.
- 5.4 ~~The student will find the sum, difference, and product of two numbers expressed as decimals through thousandths, [Moved to new SOL 5.4 b.] using an appropriate method of calculation, including paper and pencil, estimation, mental computation, and calculators. [Move to Curriculum Framework]~~
- 5.5 The student, given a dividend of four digits or fewer and a divisor of two digits or fewer, will find the quotient and remainder.
- 5.6 The student, given a dividend expressed as a decimal through thousandths and a single-digit divisor, will find the quotient.
- 5.7 The student will add and subtract with fractions and mixed numbers, ~~with and without regrouping,~~ and express answers in simplest form. ~~Problems will include like and unlike denominators limited to 12 or less.~~ [Move to Curriculum Framework]
- 5.8 The student will apply the rules for the order of operations limited to positive whole numbers [Move to Curriculum Framework] including parentheses, addition, subtraction, multiplication, and division to solve problems.

Measurement

(Focus: Perimeter, Area, Volume, and Equivalent Measures)

- 5.8 5.9 The student will
- a) describe and determine the ~~find~~ perimeter, of a polygon and the area, and volume of a square, rectangle, and right triangle, given the appropriate measures, in standard units of measure;
 - b) differentiate between perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation;
 - c) identify equivalent measurements within the metric system;
 - d) estimate and then measure to solve problems using U.S. Customary and metric units; and
 - e) choose an appropriate unit of measure for a given situation involving measurement using U.S. Customary and metric units.
- 5.9 5.10 The student will identify and describe the diameter, radius, chord, and circumference of a circle.
- 5.10 ~~The student will differentiate between perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation.~~ [Moved to new SOL 5.9 b]
- 5.11 ~~The student will choose an appropriate measuring device and unit of measure to solve problems involving measurement of~~ [Moved to new SOL 5.9 e]
- a) ~~length — part of an inch ($\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$), inches, feet, yards, miles, millimeters, centimeters, meters, and kilometers;~~ [Moved to new SOL 5.9 d]
 - b) ~~weight/mass — ounces, pounds, tons, grams, and kilograms;~~ [Moved to new SOL 5.9 d]
 - c) ~~liquid volume — cups, pints, quarts, gallons, milliliters, and liters;~~ [Moved to new SOL 5.9 d]
 - d) ~~area — square units; and~~ [Moved to new SOL 5.9 d]
 - e) ~~temperature — Celsius and Fahrenheit units.~~ [Moved to new SOL 5.9 d]
- ~~Problems also will include estimating the conversion of Celsius and Fahrenheit units relative to familiar situations (water freezes at 0°C and 32°F , water boils at 100°C and 212°F , normal body temperature is about 37°C and 98.6°F).~~ [Move to Curriculum Framework]
- 5.12 5.11 The student will determine an amount of elapsed time in hours and minutes within a 24-hour period.
- 5.13 5.12 The student will measure ~~and draw~~ right, acute, ~~and obtuse,~~ and straight angles ~~and triangles, using appropriate tools.~~ [Move to Curriculum Framework]

Geometry

(Focus: Classification and Subdividing)

- ~~5.14~~ 5.13 The student will classify
- a) angles and triangles as right, acute, or obtuse, or straight; and
 - b) triangles as right, acute, obtuse, equilateral, scalene, or isosceles.
- ~~5.15~~ 5.14 The student, using ~~two-dimensional (plane)~~ figures, (square, rectangle, triangle, parallelogram, rhombus, kite, and trapezoid) will
- a) ~~recognize, identify, describe, and analyze their properties in order to develop definitions of these plane figures; and~~
 - b) ~~identify and explore congruent, nonecongruent, [Moved to new SOL 3.16] and similar figures; [Moved to new SOL 7.6]~~
 - e) b) investigate and describe the results of combining and subdividing shapes plane figures;
 - d) ~~identify and describe a line of symmetry; and [Moved to new SOL 2.15]~~
 - e) ~~recognize the images of figures resulting from geometric transformations such as translation (slide), reflection (flip), or rotation (turn). [Moved to new SOL 4.11 b]~~
- ~~5.16~~ The student will identify, compare, and analyze properties of three-dimensional (solid) geometric shapes (cylinder, cone, cube, square pyramid, and rectangular prism). [Moved to new SOL 3.14]

Probability and Statistics

(Focus: Outcomes and Measures of Center)

- ~~5.17~~ 5.15 The student will
- a) make predictions and solve problems involving the probability of a single event an outcome by using tree diagrams or by constructing a sample space representing all possible results; and
 - b) ~~predict the probability of outcomes of simple experiments, representing it with fractions or decimals from 0 to 1, and test the prediction; [Moved to new SOL 4.13 c] and~~
 - e) b) create a problem probability statement involving probability and based on information from a given problem situation. Students will not be required to solve the created problem statement. [Move to Curriculum Framework]
- ~~5.18~~ 5.16 The student will, given a problem situation, collect, organize, and display a set of numerical data in a variety of forms, using bar graphs, stem-and-leaf plots, and line graphs, to draw conclusions and make predictions.
- ~~5.19~~ 5.17 The student will
- a) describe mean, median, and mode as measures of center;
 - b) find the mean, median, mode, and range of a set of data;
 - c) describe the range of a set of data as a measure of variation; and
 - d) describe mean as fair share.

Patterns, Functions, and Algebra

(Focus: Equations and Properties)

- ~~5.20~~ 5.18 The student will ~~analyze the structure of numerical and geometric~~ describe the relationship found in a number patterns (~~how they change or grow~~) and express the relationship, ~~using words, tables, graphs, or a mathematical sentence. Concrete materials and calculators will be used.~~ [Move to Curriculum Framework]
- ~~5.21~~ 5.19 The student will
- a) investigate and describe the concept of variable;
 - b) ~~use a variable expression to represent a given verbal quantitative expression involving one operation; and~~
 - e) b) write an open sentence to represent a given mathematical relationship, using a variable;
 - c) model one-step linear equations in one variable using addition and subtraction; and
 - d) create a problem situation based on a given open sentence using a single variable.
- ~~5.22~~ The student will ~~create a problem situation based on a given open sentence using a single variable.~~ [Moved to new SOL 5.19 d]
- 5.20 The student will investigate and recognize the distributive property of multiplication over addition.

Grade Six

~~The sixth-grade standards place continued emphasis on the study of whole numbers, decimals, and rational numbers (fractions). Students will use ratios to compare data sets; make conversions within a given measurement system; classify three-dimensional figures; collect, analyze, display, and interpret data, using a variety of graphical and statistical methods; begin using integers and percents; find the probability of an event; and investigate numerical and geometric patterns. Students will be introduced to algebraic terms and solving algebraic equations in one variable.~~

The sixth-grade standards are a transition from the emphasis placed on whole number arithmetic in the elementary grades to foundations of algebra. The standards emphasize rational numbers. Students will use ratios to compare data sets; recognize decimals, fractions and percents as ratios; solve single and multistep problems using rational numbers; and gain a foundation in the understanding of integers. Students will solve linear equations and use algebraic terminology. Students will solve problems involving area, perimeter and surface area, work with π (pi), and focus on the relationships among the properties of quadrilaterals. In addition, students will focus on applications of probability and statistics.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies ~~fraction~~ calculators, computers, and spreadsheets, ~~laser discs, and videos~~. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Students will also identify real-life applications of the mathematical principles they are learning and apply these to science and other disciplines they are studying.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense

(Focus: Relationships among Fractions, Decimals and Percents)

~~6.1 The student will identify representations of a given percent and describe orally and in writing the equivalence relationships among fractions, decimals, and percents.~~
[Moved to new SOL 6.2 b and c]

~~6.2~~ 6.1 The student will describe and compare ~~two sets of~~ data, using ratios, and will use appropriate notations, such as a/b , a to b , and $a:b$.

- 6.2 The student will
 a) investigate and describe fractions, decimals and percents as ratios;
 b) identify a given fraction, decimal, or percent from a representation;
 c) demonstrate equivalent relationships among fractions, decimals, and percents; and
 d) compare and order fractions, and decimals, and percents.
- 6.3 The student will
 a) find common multiples and factors, including least common multiple and greatest common factor; [Moved to new SOL 5.3 a]
 b) identify and describe prime and composite numbers; and identify and describe the characteristics of even and odd integers. [Moved to new SOL 5.3 b]
- 6.4 The student will compare and order whole numbers, [Moved to new SOL 3.1 c] fractions, and decimals, [Moved to new SOL 5.2 b and new SOL 6.2] using concrete materials, drawings or pictures, and mathematical symbols. [Move to Curriculum Framework]
- 6.5 6.3 The student will
 a) identify; and represent integers; ;
 b) order; and compare integers; and
 c) identify and describe absolute value of integers.
- 6.4 The student will demonstrate multiple representations of multiplication and division of fractions.
- 6.5 The student will investigate and describe concepts of positive exponents and perfect squares.

Computation and Estimation

(Focus: Applications of Operations with Rational Numbers)

- 6.6 The student will
 a) solve problems that involve addition, subtraction, multiplication, multiply and/or division divide with fractions and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less, and express their answers in simplest form [Moved to new SOL 6.6 b]; and
 b) find the quotient, given a dividend expressed as a decimal through thousandths and a divisor expressed as a decimal to thousandths with exactly one non-zero digit estimate solutions and then solve single and multistep practical problems that involve addition, subtraction, multiplication and division of fractions.
- 6.7 The student will use estimation strategies to solve single and [Added for clarity] multistep practical problems involving whole numbers, decimals, and fractions (rational numbers). addition, subtraction, multiplication and division of decimals. [Fractions moved to new SOL 6.6 a; whole numbers moved to new SOL 5.4 a]

6.8 ~~The student will solve multistep consumer application problems involving fractions and decimals and present data and conclusions in paragraphs, tables, or graphs. Planning a budget will be included. [Move to Curriculum Framework]~~

6.8 The student will apply order of operations in solving equations.

Measurement

(Focus: Problem Solving with Area, Perimeter, Volume and Surface Area)

6.9 ~~The student will compare and convert units of measure for length, area, weight/mass, and volume within the U.S. Customary system and the metric system and estimate conversions between units in each system: [Move to Curriculum Framework]~~

- ~~a) length — part of an inch (1/2, 1/4, and 1/8), inches, feet, yards, miles, millimeters, centimeters, meters, and kilometers [Move to Curriculum Framework];~~
- ~~b) weight/mass — ounces, pounds, tons, grams, and kilograms; [Move to Curriculum Framework]~~
- ~~c) liquid volume — cups, pints, quarts, gallons, milliliters, and liters; and [Move to Curriculum Framework]~~
- ~~d) area — square units. * [Move to Curriculum Framework]~~

** The intent of this standard is for students to make ballpark comparisons and not to memorize conversion factors between U.S. Customary and metric units. make ballpark comparisons between the U.S. Customary System of measurement and the metric system.*

6.10 ~~The student will estimate and then determine length, weight/mass, area, and liquid volume/capacity, using standard and nonstandard units of measure. [Moved to new SOL 5.9 d]~~

6.10 The student will

- a) define pi (π) as the ratio of the circumference of a circle to its diameter;
- b) solve practical problems involving circumference and area of a circle;
- c) solve practical problems involving area and perimeter; and
- d) describe and determine the volume and surface area of a rectangular prism.

6.11 ~~The student will determine if a problem situation involving polygons of four or fewer sides represents the application of perimeter or area and apply the appropriate formula. [Moved to new SOL 5.9 b]~~

6.12 ~~The student will~~

- ~~a) solve problems involving the circumference and/or area of a circle when given the diameter or radius; and [Moved to new SOL 6.10 b]~~
- ~~b) derive approximations for pi (π) from measurements for circumference and diameter, [Moved to new SOL 6.10 a] using concrete materials or computer models. [Move to Curriculum Framework]~~

- 6.13 ~~The student will~~
- a) ~~estimate angle measures, using 45° , 90° , and 180° as referents, and use the appropriate tools to measure the given angles; and~~
 - b) ~~measure and draw right, acute, and obtuse angles and triangles. [Moved to new SOL 5.12 and SOL 5.13]~~

Geometry

(Focus: Properties and Relationships)

- 6.11 The student will
- a) identify the coordinates of a point in a coordinate plane; and
 - b) graph ordered pairs in a coordinate plane.
- 6.14 ~~The student will identify, classify, and describe the characteristics of plane figures, describing their similarities, differences, and defining properties. [Moved to new SOL 6.13]~~
- 6.15 6.12 ~~The student will determine congruence of segments, angles, and polygons by direct comparison, given their attributes. Examples of noncongruent and congruent figures will be included. [Move to Curriculum Framework]~~
- 6.16 ~~The student will construct the perpendicular bisector of a line segment and an angle bisector.~~
- 6.17 ~~The student will sketch, construct models of, and classify solid figures (rectangular prism, cone, cylinder, and pyramid).~~
- 6.13 The student will describe and identify properties of quadrilaterals.

Probability and Statistics

(Focus: Practical Applications of Statistics)

- 6.18 6.14 ~~The student, given a problem situation, will collect, analyze, display, and interpret data in a variety of graphical methods, including~~
- a) ~~line, [Moved to new SOL 5.16] bar, and construct circle graphs and histograms;~~
 - b) ~~stem-and-leaf plots; [Moved to new SOL 5.16] and draw conclusions and make predictions using circle graphs and histograms; and~~
 - c) ~~box-and-whisker plots. [Moved to new SOL 7.10 a] compare graphs which present the same information.~~
- ~~Circle graphs will be limited to halves, fourths, and eighths. [Move to Curriculum Framework]~~

- ~~6.19~~ 6.15 The student will
- a) ~~describe the mean, median, and mode as measures of central tendency~~ [Moved to new SOL 5.17 a] ; balance point;
 - b) ~~describe the range, and determine their meaning for a set of data.~~ [Moved to new SOL 5.17 c] decide which measure of center is appropriate for a given situation.
- ~~6.20~~ 6.16 The student will
- a) ~~make a sample space for selected experiments and represent it in the form of a list, chart, picture, or tree diagram;~~ [Moved to new SOL 5.16] distinguish between dependent and independent events; and
 - b) ~~determine and interpret the probability of an event occurring from a given sample space and represent the probability as a ratio, decimal or percent, as appropriate for the given situation.~~ probabilities for dependent and independent events.

Patterns, Functions, and Algebra

(Focus: Variable Equations and Properties)

- ~~6.21~~ 6.17 The student will ~~investigate, describe, identify~~ and extend ~~numerical and geometric patterns, including triangular numbers, patterns formed by powers of 10;~~ [Move to Curriculum Framework] and arithmetic sequences.
- ~~6.22~~ The student will ~~investigate and describe concepts of positive exponents, perfect squares;~~ [Moved to new SOL 6.5] ~~square roots, and, for numbers greater than 10, scientific notation.~~ [Moved to new SOL 7.1] ~~Calculators will be used to develop exponential patterns.~~ [Move to Curriculum Framework]
- ~~6.23~~ 6.18 The student will
- a) ~~model and solve algebraic equations, using concrete materials;~~ [Moved to new SOL 5.19 c]
 - b) ~~solve one-step linear equations in one variable, involving whole number coefficients and positive rational solutions; and~~
 - c) ~~use the following algebraic terms appropriately: variable, coefficient, term, and equation.~~
- 6.19 The student will investigate and recognize:
- a) the identity properties for addition and multiplication;
 - b) the multiplicative property of zero; and
 - c) the inverse properties for addition and multiplication.
- 6.20 The student will graph inequalities on a number line.

Grade Seven

The seventh-grade standards place emphasis on solving problems involving consumer applications, using proportional reasoning, and gaining proficiency in computations with integers. The students will gain an understanding of the properties of real numbers, solve one-step linear equations and inequalities, and use data analysis techniques to make inferences, conjectures, and predictions. Two- and three-dimensional representations, graphing transformations in the coordinate plane, and probability will be extended.

The seventh-grade standards continue to emphasize the foundations of algebra. Students who successfully complete the seventh-grade standards should be prepared to study Algebra I in grade eight. Topics in grade seven include proportional reasoning, integer computation, solving two-step linear equations, and recognizing different representations for relationships. Students will apply the properties of real numbers in solving equations, solve inequalities, and use data analysis techniques to make inferences, conjectures, and predictions.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies such as ~~fraction~~ calculators, computers, and spreadsheets, ~~laser discs, and videos~~. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Students will also identify real-life applications of the mathematical principles they are learning and apply these to science and other disciplines they are studying.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense (Focus: Scientific Notation and Square Roots)

- 7.1 The student will
- investigate and describe the concept of negative exponents; compare, order, and determine equivalent relationships between fractions, decimals, and percents;
 - including use of determine scientific notation for numbers greater than 10 zero;
 - compare and order numbers written in scientific notation;
 - determine square roots; and
 - identify and describe absolute value for rational numbers.

- 7.2 The student will simplify expressions that contain rational numbers (whole numbers, fractions, and decimals) and positive exponents, using order of operations, [Moved to new SOL 6.8] mental mathematics, and appropriate tools. [Move to Curriculum Framework]
- 7.3 The student will identify and apply the following properties of operations with real numbers:
- a) ~~the commutative and associative properties for addition and multiplication;~~
[Moved to new SOL 3.20 a and new SOL 4.16]
 - b) ~~the distributive property;~~ [Moved to new SOL 5.20]
 - c) ~~the additive and multiplicative identity properties;~~ [Moved to new SOL 6.19 a]
 - d) ~~the additive and multiplicative inverse properties;~~ and [Moved to new SOL 6.19 c]
 - e) ~~the multiplicative property of zero.~~ [Moved to new SOL 6.19 b]

Computation and Estimation

(Focus: Application of Rational Number Operations and Proportional Reasoning)

- 7.4 The student will
- a) ~~solve practical problems using rational numbers (whole numbers, fractions, decimals) and percents; and~~ [Moved to new SOL 7.3]
 - b) ~~solve consumer application problems involving tips, discounts, sales tax, and simple interest.~~ [Move to Curriculum Framework]
- 7.5 ~~7.2~~ The student will
- a) ~~formulate rules for model addition, subtraction, multiplication and division of integers;~~ and
 - b) ~~solve practical problems involving basic operations (addition, subtraction, multiplication, and division) with integers.~~ [Moved to new SOL 7.3] add, subtract, multiply, and divide integers.
- 7.6 The student will use proportions to solve practical problems, which may include scale drawings, that contain rational numbers (whole numbers, fractions, and decimals) and percents. [Move to Curriculum Framework to support new SOL 7.3]
- 7.3 The student will solve single and multistep practical problems using rational numbers.

Measurement

(Focus: Volume and Surface Area)

- 7.7 The student, given appropriate dimensions, will
- a) ~~estimate and find the area of polygons by subdividing them into rectangles and right triangles;~~ and [Moved to new SOL 8.12.]
 - b) ~~apply perimeter and area formulas in practical situations.~~ [Moved to new SOL 6.10 c]

- ~~7.8~~ 7.4 The student will
- a) ~~investigate and~~ describe volume and surface area of cylinders; and
 - b) solve practical problems involving the volume and surface area of rectangular prisms and [Moved to new SOL 6.10 d] cylinders; ~~using concrete materials and practical situations to develop formulas.~~ [Move to Curriculum Framework]

Geometry

(Focus: Relationships Between Figures)

- ~~7.9~~ 7.5 The student will compare and contrast the following quadrilaterals based on properties: parallelogram, rectangle, square, rhombus, and trapezoid. ~~Deductive reasoning and inference will be used to classify quadrilaterals.~~ [Move to Curriculum Framework]
- ~~7.10~~ The student will identify and draw the following polygons: ~~pentagon, hexagon, heptagon, octagon, nonagon, and decagon.~~ [Moved to new SOL 4.12]
- ~~7.11~~ 7.6 The student will determine if geometric plane figures — quadrilaterals and triangles — are similar and write proportions to express the relationships between corresponding parts of similar figures.
- ~~7.12~~ The student will identify and graph ordered pairs in the four quadrants of a coordinate ~~plane.~~ [Moved to new SOL 6.11]
- ~~7.13~~ 7.7 The student, given a polygon in the coordinate plane, will represent transformations — reflections, dilations, rotations, and translations — by graphing the coordinates of the vertices of the transformed polygon and sketching the resulting figure.

Probability and Statistics

(Focus: Applications of Statistics and Probability)

- ~~7.14~~ 7.8 The student will investigate and describe the difference between the ~~probability of an event found through simulation versus~~ experimental and theoretical probability of ~~that same an~~ event.
- ~~7.15~~ 7.9 The student will ~~identify and describe the number of possible arrangements of several objects, using a tree diagram or~~ determine the probability of compound events using the Fundamental (Basic) Counting Principle.
- ~~7.16~~ The student will ~~create and solve problems involving the measures of central tendency (mean, median, mode) and the range of a set of data.~~

- ~~7.17~~ 7.10 The student, given data in a problem practical situation, will ~~collect, analyze, display, and interpret data, using a variety of graphical methods, including~~
- a) ~~frequency distributions; construct and analyze box-and-whisker plots; and~~
 - b) ~~line plots; compare data presented in box-and-whisker plots and other graphs.~~
 - c) ~~histograms; [Moved to new SOL 6.14 a]~~
 - d) ~~stem-and-leaf plots; [Moved to new SOL 5.16]~~
 - e) ~~box-and-whisker plots; [Moved to new SOL 7.10 a] and~~
 - f) ~~scattergrams.~~
- 7.18 The student will ~~make inferences, conjectures, and predictions based on analysis of a set of data.~~ [Moved to new SOL 7.10 and new SOL 8.13]

Patterns, Functions, and Algebra

(Focus: Linear Equations)

- ~~7.19~~ 7.11 The student will ~~represent, analyze, and generalize a variety of patterns, including arithmetic sequences and geometric sequences, relationships with tables, graphs, rules, and words, in order to investigate and describe functional relationships.~~
- ~~7.20~~ 7.12 The student will
- a) write verbal expressions as algebraic expressions and sentences as equations and vice versa; and
 - b) evaluate algebraic expressions.
- ~~7.21~~ The student will ~~use the following algebraic terms appropriately: equation, inequality, and expression.~~
- ~~7.22~~ 7.13 The student will
- a) solve one- ~~step~~ and two-step linear equations ~~and inequalities~~ [Moved to new SOL 7.14 a] in one variable ~~with strategies involving inverse operations and integers, using concrete materials, pictorial representations, and paper and pencil [Move to Curriculum Framework]; and~~
 - b) solve practical problems requiring the solution of a one- ~~or two-step~~ linear equation.
- 7.14 The student will
- a) solve one-step inequalities in one variable; and
 - b) graph solutions to inequalities on the number line.
- 7.15 The student will ~~identify and~~ apply the following properties of operations with real numbers:
- a) the commutative and associative properties for addition and multiplication;
 - b) the distributive property;
 - c) the additive and multiplicative identity properties;
 - d) the additive and multiplicative inverse properties; and
 - e) the multiplicative property of zero.

Grade Eight

The eighth-grade standards are intended to serve two purposes. First, the standards contain both content that reviews or extends concepts and skills learned in previous grades, and Second, they contain new content that prepares students for more abstract concepts in algebra and geometry. The eighth grade standards provide students additional instruction and time to acquire the concepts and skills necessary for success in Algebra I. Students will gain proficiency in computation with rational numbers (~~positive and negative fractions, positive and negative decimals, whole numbers, and integers~~) and use proportions to solve a variety of problems. New concepts include solving ~~two-step~~ multistep equations and inequalities, graphing linear equations, visualizing three-dimensional shapes represented in two-dimensional drawings, and applying transformations to geometric shapes in the coordinate plane, ~~and using matrices to organize and interpret data~~. Students will verify and apply the Pythagorean Theorem and represent relations and functions using tables, graphs, and rules.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies ~~such as fraction calculators, computers, spreadsheets, laser discs, and videos~~. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Students will also identify real-life applications of the mathematical principles they are learning that can be applied to science and other disciplines they are studying.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense

(Focus: Relationships within the Real Number System)

- 8.1 The student will
- a) simplify numerical expressions involving positive and negative exponents, using rational numbers, order of operations, and properties of operations with real numbers; and
 - ~~b) recognize, represent, compare, and order numbers expressed in scientific notation; and [Moved to new SOL 7.1 c]~~
 - ⇒ b) compare and order decimals, fractions, percents, and numbers written in scientific notation.
- 8.2 The student will describe orally and in writing the relationship between the subsets of the real number system.

Computation and Estimation

(Focus: Practical Applications of Operations with Real Numbers)

- 8.3 The student will solve practical problems involving rational numbers, percents, ratios, and proportions. ~~Problems will be of varying complexities and will involve real-life data, such as finding a discount and discount prices and balancing a checkbook. [Move to Curriculum Framework]~~
- 8.4 The student will determine the percent increase or decrease for a given situation.
- 8.4 8.5 The student will apply the order of operations to evaluate algebraic expressions for given replacement values of the variables. ~~Problems will be limited to positive exponents. [Move to Curriculum Framework]~~
- 8.5 8.6 The student, ~~given a whole number from 0 to 100,~~ will
- a) identify a given whole number ~~it~~ as a perfect square; and
 - b) or find the two consecutive whole numbers between which ~~the~~ a square root lies.

Measurement

(Focus: Problem Solving)

- ~~8.6~~ 8.7 The student will
- a) verify by measuring and describe the relationships among vertical angles, adjacent angles, supplementary angles, and complementary angles; and
 - b) will measure ~~and draw~~ angles of less than 360° .
- ~~8.7~~ 8.8 The student will investigate and solve practical problems involving volume and surface area of ~~rectangular solids (prisms),~~ cylinders, cones, and pyramids.

Geometry

(Focus: Problem Solving with 2- and 3-Dimensional Figures)

- ~~8.8~~ 8.9 The student will
- a) apply transformations (~~rotate or turn, reflect or flip, translate or slide, and dilate or scale~~) [Move to Curriculum Framework] to geometric plane figures; and represented on graph paper. The student will
 - b) identify applications of transformations, ~~such as tiling, fabric design, art, and sealing.~~ [Move to Curriculum Framework]
- ~~8.9~~ 8.10 The student will construct a three-dimensional model, given the top, side, and/or bottom views.
- ~~8.10~~ 8.11 The student will
- a) verify the Pythagorean Theorem, ~~using diagrams, concrete materials, and measurement;~~ and [Move to Curriculum Framework]
 - b) apply the Pythagorean Theorem and its converse ~~to find the missing length of a side of a right triangle when given the lengths of the other two sides.~~ [Move to Curriculum Framework]
- 8.12 The student will solve practical area and perimeter problems involving composite, plane figures.

Probability and Statistics

(Focus: Statistical Analysis of Graphs and Problem Situations)

- ~~8.11~~ 8.13 The student will analyze problem situations, ~~including games of chance, board games, or grading scales,~~ [Move to Curriculum Framework] and make predictions, using knowledge of probability.
- ~~8.12~~ 8.14 The student will
- a) make comparisons, predictions, and inferences, using information displayed in ~~frequency distributions; box and whisker plots; scattergrams; line, bar, circle, and picture graphs; and histograms~~ [Move to Curriculum Framework] graphs; and
 - b) construct and analyze scatterplots.
- ~~8.13~~ The student will ~~use a matrix to organize and describe data.~~

Patterns, Functions, and Algebra

(Focus: Linear Relationships)

- 8.14 ~~8.15~~ The student will
- a) ~~describe and represent a given relationship relations and functions, using in tables, graphs, word, and rules form; and~~
 - b) ~~relate and compare tables, graphs, and rules as different forms of representation for relationships. make connections between any two forms.~~
- 8.15 ~~8.16~~ The student will
- a) ~~solve two- multistep linear equations and inequalities in one with variables; on one or two sides of the equation using concrete materials, pictorial representations, and paper and pencil; [Move to Curriculum Framework]~~
 - b) solve two-step inequalities and graph the results on a number line; and
 - c) identify properties of operations used to solve an equation.
- 8.16 ~~8.17~~ The student will graph a linear equation in two variables, ~~in the coordinate plane,~~ using a table of ordered pairs.
- 8.17 The student will ~~create and solve problems, using proportions, formulas, and functions. [Moved to new SOL 8.3]~~
- 8.18 The student will ~~use the following algebraic terms appropriately: domain, range, independent variable, and dependent variable. identify the domain, range, independent variable or dependent variable in a given situation.~~

Algebra I

The standards below outline the content for a one-year course in Algebra I. All students are expected to achieve the Algebra I standards. When planning for instruction, consideration will be given to the sequential development of concepts and skills by using concrete materials to assist students in making the transition from the arithmetic to the symbolic. Students should be helped to make connections and build relationships between algebra and arithmetic, geometry, and probability and statistics. Connections also should be made to other subject areas through practical applications. This approach to teaching algebra should help students attach meaning to the abstract concepts of algebra.

These standards require students to use algebra as a tool for representing and solving a variety of practical problems. Tables and graphs will be used to interpret algebraic expressions, equations, and inequalities and to analyze functions behavior. ~~Matrices will be used to organize and manipulate data.~~

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of functions; they provide a powerful tool for solving and verifying solutions to equations and inequalities.

Throughout the course, students should be encouraged to ~~talk~~ engage in discourse about mathematics with teachers and other students, use the language and symbols of mathematics in representations and communication, discuss problems and problem solving, and develop ~~their~~ confidence in themselves as mathematics students.

Expressions and Operations

- A.1 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.
- A.2 ~~The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.~~ [Moved to new SOL A.1] ~~Students will choose an appropriate computational technique, such as mental mathematics, calculator, or paper and pencil.~~ [Move to Curriculum Framework]
- A.2 The student will perform operations on polynomials including:
- a) applying the laws of exponents to perform operations on expressions;
 - b) adding, subtracting, multiplying, and dividing polynomials; and
 - c) factoring completely first- and second-degree binomials and trinomials in one or two variables.
- A.10 ~~The student will apply the laws of exponents to perform operations on expressions~~ [Moved to new SOL A.2 a] ~~with integral exponents, using scientific notation when appropriate.~~ [Move to Curriculum Framework]

- A.11 ~~The student will add, subtract, and multiply polynomials and divide polynomials [Moved to new SOL A.2 b] with monomial divisors using concrete objects, pictorial and area representations, and algebraic manipulations. [Move to Curriculum Framework]~~
- A.12 ~~The student will factor completely first- and second-degree binomials and trinomials in one or two variables. [Moved to new SOL A.2 c] The graphing calculator will be used as a tool for factoring and for confirming algebraic factorizations. [Move to Curriculum Framework]~~
- A.13 A.3 The student will express the square root and cube root of whole numbers and the square root of a monomial algebraic expression in simplest radical form, ~~and approximate square roots to the nearest tenth.~~

Equations and Inequalities

- A.4 The student will solve multistep linear and quadratic equations ~~and inequalities in one two variables, including:~~
- a) solving literal equations (formulas) for a given variable, and ;
 - b) justifying steps used in simplifying expressions and solving equations and inequalities using field properties, axioms of equality and inequality, and properties of order that are valid for the set of real numbers and its subsets;
 - c) solving quadratic equations in one variable both algebraically and graphically;
 - d) solving multistep linear equations algebraically and graphically;
 - e) solving systems of two linear equations in two variables both algebraically and graphically;
 - f) apply solving real-world problems involving equations and systems of equations.
- A.5 The student will solve multistep linear equations ~~and inequalities in one two variables, including:~~
- a) solving multistep linear inequalities algebraically and graphically; and
 - b) solving real-world problems involving inequalities.
- A.1 ~~The student will solve multistep linear equations [Moved to new SOL A.4 d] and inequalities in one variable; [Moved to new SOL A.5] solve literal equations (formulas) for a given variable; [Moved to new SOL A.4 a] and apply these skills to solve practical problems. [Moved to new SOL A.4 f] Graphing calculators will be used to confirm algebraic solutions. [Move to Curriculum Framework]~~
- A.3 ~~The student will justify steps used in simplifying expressions and solving equations and inequalities. [Moved to new SOL A.4 b] Justifications will include the use of concrete objects; pictorial representations; and the properties of real numbers, equality, and inequality. [Move to Curriculum Framework]~~

- A.9 ~~The student will solve systems of two linear equations in two variables both algebraically and graphically and apply these techniques to solve practical problems. [Moved to new SOL A.4 e] Graphing calculators will be used both as a primary tool for solution and to confirm an algebraic solution. [Move to Curriculum Framework]~~
- A.14 ~~The student will solve quadratic equations in one variable both algebraically and graphically. [Moved to new SOL A.4 c] Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions. [Move to Curriculum Framework]~~
- A.6 ~~The student will select, justify, and apply an appropriate technique to graph linear functions equations and linear inequalities in two variables. Techniques will include slope intercept, x and y intercepts, graphing by transformation, and the use of the graphing calculator. [Move to Curriculum Framework], including:~~
- ~~a) determineing the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined;~~
 - ~~b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line; and~~
 - ~~c) graphing linear functions and inequalities in two variables.~~
- A.6 ~~The student will [Moved to new SOL A.6], select justify, and apply an appropriate technique to [Move to Curriculum Framework] graph linear functions and linear inequalities in two variables. [Moved to new SOL A.6 c] Techniques will include slope intercept, x and y intercepts, graphing by transformation, and the use of the graphing calculator. [Move to Curriculum Framework]~~
- A.7 ~~The student will determine the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined. [Moved to new SOL A.6 a] The graphing calculator will be used to investigate the effect of changes in the slope on the graph of the line. [Move to Curriculum Framework]~~
- A.8 ~~The student will write an equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line. [Moved to new SOL A.6 b]~~

Functions

- A.7 The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including:
- a) determining whether a relation is a function;
 - b) domain and range;
 - c) zeros of a function;
 - d) x- and y-intercepts;
 - e) intervals in which the function is increasing/decreasing;
 - f) finding the values of a function for elements in its domain; and
 - g) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.
- ~~A.5~~ ~~The student will create and use tabular, symbolic, graphical, verbal, and physical representations [Moved to new SOL A.7 g.] to analyze a given set of data for the existence of a pattern, determine the domain and range of relations, [Moved to new SOL A.7 b.] and identify the relations that are functions. [Moved to new SOL A.7 a.]~~
- ~~A.15~~ ~~The student will, given a rule, find the values of a function for elements in its domain [Moved to new SOL A.7 f.] and locate the zeros of the function [Moved to new SOL A.7 c.] both algebraically and with a graphing calculator. The value of $f(x)$ will be related to the ordinate on the graph. [Move to Curriculum Framework]~~
- ~~A.18~~ A.8 The student will, given a situation in a real-world context, analyze a relation to determine whether a direct variation exists, and represent it algebraically and graphically, if possible, and inverse variation algebraically.

Statistics

- ~~A.4~~ ~~The student will use matrices to organize and manipulate data, including matrix addition, subtraction, and scalar multiplication. Data will arise from business, industrial, and consumer situations.~~
- ~~A.16~~ A.8 The student will, given a set of data points, write an equation for a line collect and analyze data, determine the equation of the curve of best fit use the equation in order to make predictions, and solve real-world problem using mathematical models. Mathematical models will include linear and quadratic functions.
- ~~A.17~~ A.10 The student will, given a set of data, compare and contrast multiple one-variable data sets, using statistical techniques that include measures of central tendency and range, interpret variation in real-world contexts and calculate and interpret mean deviation, standard deviation, and z-scores.

Geometry

This course is designed for students who have successfully completed the standards for Algebra I. All students are expected to achieve the Geometry standards. The course includes, among other things, properties of geometric figures, trigonometric relationships, and reasoning to justify conclusions. Methods of justification will include paragraph proofs, two-column proofs, indirect proofs, coordinate proofs, algebraic methods, and verbal arguments. A gradual development of formal proof is encouraged. Inductive and intuitive approaches to proof as well as deductive axiomatic methods should be used.

This set of standards includes emphasis on two- and three-dimensional reasoning skills, coordinate and transformational geometry, and the use of geometric models to solve problems. A variety of applications and some general problem-solving techniques, including algebraic skills, should be used to implement these standards. Calculators, computers, graphing utilities (graphing calculators or computer graphing simulators), dynamic geometry software, and other appropriate technology tools will be used to assist in teaching and learning. Any technology that will enhance student learning should be used.

Reasoning, Lines, and Transformations

- G.1 The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
- identifying the converse, inverse, and contrapositive of a conditional statement;
 - translating a short verbal argument into symbolic form;
 - using Venn diagrams to represent set relationships; and
 - using deductive reasoning, ~~including the law of syllogism.~~ [Move to Curriculum Framework]
- ~~G.4~~ G.2 The student will use the relationships between angles formed by two lines cut by a transversal to:
- determine if two lines are parallel; and
 - verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
 - solve real-world problems involving angles formed when parallel lines are cut by a transversal.
- ~~G.3~~ ~~The student will solve practical problems involving complementary, supplementary, and congruent angles that include vertical angles, [Moved to new SOL 8.7 a] angles formed when parallel lines are cut by a transversal, [Moved to new SOL G.2] and angles in polygons. [Moved to new SOL G.10]~~

- ~~G-2~~ G.3 The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include:
- a) investigating and using formulas for finding distance, midpoint, and slope;
 - b) applying slope to verify and determine if lines are parallel or perpendicular;
 - ~~b)~~ c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
 - ~~e)~~ d) determining whether a figure has been translated, reflected, ~~or~~ rotated, or dilated using coordinate methods.
- ~~G-11~~ G.4 The student will construct and justify the constructions of:
- a) a line segment congruent to a given line segment;
 - b) the perpendicular bisector of a line segment;
 - c) a perpendicular to a given line from a point not on the line;
 - d) a perpendicular to a given line ~~at~~ through a given point on the line;
 - e) the bisector of a given angle; ~~and~~;
 - f) an angle congruent to a given angle; and
 - g) a line parallel to a given line through a point not on the given line;

Triangles

- ~~G-6~~ G.5 The student ~~will~~, given information concerning the lengths of sides and/or measures of angles in triangles, will ~~apply the triangle inequality properties to determine whether a triangle exists and to order sides and angles.~~
- a) order the sides by length, given the angle measures;
 - b) order the angles by degree measure, given the side lengths;
 - c) determine whether a triangle exists; and
 - d) determine the range in which the length of the third side must lie.
- These concepts will be considered in the context of ~~practical~~ real-world situations.
- ~~G-5~~ G.6 The student will
- a) ~~investigate and identify congruence and similarity relationships between triangles; and~~
 - ~~b)~~ prove two triangles are congruent ~~or similar~~, [Moved to new SOL G.7] given information in the form of a figure or statement, using algebraic and coordinate methods as well as deductive proofs.
- G.7 The student will prove two triangles are similar given information in the form of a figure or statement, using algebraic and coordinate methods as well as deductive proofs.
- ~~G-7~~ G.8 The student will solve ~~practical~~ real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry. ~~Solutions will be expressed in radical form or as decimal approximations. [Move to Curriculum Framework]~~

Polygons and Circles

- G.8 G.9 The student will
- a) ~~investigate and identify properties of quadrilaterals involving opposite sides and angles, consecutive sides and angles, and diagonals;~~
 - b) ~~prove these properties of quadrilaterals, using algebraic and coordinate methods as well as deductive reasoning;~~ [Move to Curriculum Framework] and
 - e) use properties of quadrilaterals to solve practical real-world problems.
- G.10 The student will solve real-world problems involving angles of polygons.
- G.9 ~~The student will use measures of interior and exterior angles of polygons to solve problems. [Moved to new SOL G.10] Tessellations and tiling problems will be used to make connections to art, construction, and nature. [Move to Curriculum Framework]~~
- G.11 The student will use angles, arcs, chords, tangents, and secants to:
- a) investigate, prove, and apply properties of circles;
 - b) solve real-world problems involving properties of circles; and
 - c) find arc length and areas of sectors in circles.
- G.10 ~~The student will investigate and solve practical problems using properties of angles, arcs, chords, tangents, and secants. Problems will include finding arc length and the area of a sector, and may be drawn from applications of architecture, art, and construction. [Move to Curriculum Framework]~~

Three-Dimensional Figures

- G.12 ~~The student will make a model of a three-dimensional figure from a two-dimensional drawing and make a two-dimensional representation of a three-dimensional object. Models and representations will include scale drawings, perspective drawings, blueprints, or computer simulations.~~
- G.13 G.12 The student will use formulas for surface area and volume of three-dimensional objects to solve practical real-world problems. ~~Calculators will be used to find decimal approximations for results. [Move to Curriculum Framework]~~
- G.14 G.13 The student will use similar geometric objects in two- or three-dimensions to:
- a) ~~use proportional reasoning to solve practical problems, given similar geometric objects; and compare ratios between side lengths, perimeters, areas, and volumes;~~
 - b) determine how changes in one or more dimensions of an object affect area and/or volume of the object; and
 - c) solve real-world problems about similar geometric objects.

Algebra, Functions, and Data Analysis

[Adopted by the Virginia Board of Education on June 28, 2007]

The following standards outline the content for a one-year course in Algebra, Functions, and Data Analysis. All students who are pursuing a technical field are expected to achieve the Algebra, Functions, and Data Analysis or Algebra II standards. This course is designed for students who have successfully completed the standards for Algebra I. Within the context of mathematical modeling and data analysis, students will study functions and their behaviors, systems of inequalities, probability, experimental design and implementation, and analysis of data. Data will be generated by practical applications arising from science, business, and finance. Students will solve problems that require the formulation of linear, quadratic, exponential, or logarithmic equations or a system of equations.

Through the investigation of mathematical models and interpretation/analysis of data from real-life situations, students will strengthen conceptual understandings in mathematics and further develop connections between algebra and statistics. Students should use the language and symbols of mathematics in representations and communication throughout the course.

These standards include a transformational approach to graphing functions and writing equations when given the graph of the equation. Transformational graphing builds a strong connection between algebraic and graphic representations of functions.

The infusion of technology (graphing calculator and/or computer software) in this course will assist in modeling and investigating functions and data analysis.

Algebra and Functions

- AFDA.1 The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include:
- a) continuity;
 - b) local and absolute maxima and minima;
 - c) domain and range;
 - d) zeros;
 - e) intercepts;
 - f) intervals in which the function is increasing/decreasing;
 - g) end behaviors; and
 - h) asymptotes.
- AFDA.2 The student will use knowledge of transformations to write an equation given the graph of a function (linear, quadratic, exponential, and logarithmic).

- AFDA.3 The student will collect data and generate an equation for the curve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world problems or applications. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.
- AFDA.4 The student will transfer between and analyze multiple representations of functions including algebraic formulae, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.
- AFDA.5 The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques.

Data Analysis

- AFDA.6 The student will calculate probabilities. Key concepts include:
- a) conditional probability;
 - b) dependent and independent events;
 - c) addition and multiplication rules;
 - d) counting techniques (permutations and combinations); and
 - e) Law of Large Numbers.
- AFDA.7 The student will analyze the normal distribution. Key concepts include:
- a) characteristics of normally distributed data;
 - b) percentiles;
 - c) normalizing data using z-scores; and
 - d) area under the standard normal curve and probability.
- AFDA.8 The student will design and conduct an experiment/survey. Key concepts include:
- a) sample size;
 - b) sampling technique;
 - c) controlling sources of bias and experimental error;
 - d) data collection; and
 - e) data analysis and reporting.

Algebra II

The standards below outline the content for a one-year course in Algebra II. Students enrolled in Algebra II are assumed to have mastered those concepts outlined in the Algebra I standards. All students preparing for postsecondary and advanced technical studies are expected to achieve the Algebra II standards. A thorough treatment of advanced algebraic concepts is provided through the study of functions, “families of functions,” equations, inequalities, systems of equations and inequalities, polynomials, rational and radical expressions equations, complex numbers, ~~matrices~~, and sequences and series. Emphasis will be placed on practical applications and modeling throughout the course of study. Oral and written communication concerning the language of algebra, logic of procedures, and interpretation of results also should permeate the course.

These standards include a transformational approach to graphing functions. Transformational graphing uses translation, reflection, dilation, and rotation to generate a “family of graphs” from a given graph and builds a strong connection between algebraic and graphic representations of functions. Students will vary the coefficients and constants of an equation, observe the changes in the graph of the equation, and make generalizations that can be applied to many graphs.

Graphing utilities (graphing calculators or computer graphing simulators), computers, spreadsheets, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through mathematical modeling and aid in the investigation and study of functions. They also provide an effective tool for solving/verifying equations and inequalities. Any other available technology that will enhance student learning should be used.

Expressions and Operations

AII.1 ~~The student will identify field properties, axioms of equality and inequality, and properties of order that are valid for the set of real numbers and its subsets, [Moved to new SOL A.4 b] complex numbers, [Moved to new SOL AII.3] and matrices.~~

AII.2 AII.1 The student will, given rational, radical, or polynomial expressions,
 a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
 b) add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables, and expressions containing rational exponents;
 c) write radical expressions as expressions containing rational exponents and vice versa; and
 d) factor ~~completely~~ polynomials completely.

AII.2 ~~The student will add, subtract, multiply, divide, and simplify rational expressions, [Moved to new SOL AII.1 a] including complex fractions. [Move to Curriculum Framework]~~

- ~~AII.3 The student will~~
~~a) add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables and expressions containing rational exponents; [Moved to new SOL AII.1 b] and~~
~~b) write radical expressions as expressions containing rational exponents and vice versa. [Moved to new SOL AII.1 c]~~
- ~~AII.5 The student will identify and factor completely polynomials [Moved to new SOL AII.1 d] representing the difference of squares, perfect square trinomials, the sum and difference of cubes, and general trinomials. [Move to Curriculum Framework]~~
- ~~AII.16~~ AII.2 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve ~~practical~~ real-world problems, including writing the first n terms, finding the n^{th} term, and evaluating summation formulas. Notation will include Σ and a_n .
- ~~AII.17~~ AII.3 The student will perform operations on complex numbers ~~and~~, express the results in simplest form. ~~Simplifying results will involve using patterns of the powers of i .~~ and identify field properties that are valid for the complex numbers.

Equations and Inequalities

- AII.4 The student will solve, algebraically and graphically,
 a) absolute value equations and inequalities;
 b) quadratic equations over the set of complex numbers;
 c) equations containing rational algebraic expressions; and
 d) equations containing radical expressions.
- ~~AII.4 The student will solve absolute value equations and inequalities [Moved to new SOL AII.4 a] graphically and algebraically. [Moved to new SOL AII.4] Graphing calculators will be used as a primary method of solution and to verify algebraic solutions. [Move to Curriculum Framework]~~
- ~~AII.6 The student will select, justify, and apply a technique to solve a quadratic equation over the set of complex numbers. [Moved to new SOL AII.4 b] Graphing calculators will be used for solving and for confirming the algebraic solutions. [Move to Curriculum Framework]~~
- ~~AII.7 The student will solve equations containing rational expressions [Moved to new SOL AII.4 c] and equations containing radical expressions algebraically and graphically. [Moved to new SOL AII.4 d] Graphing calculators will be used for solving and for confirming the algebraic solutions. [Move to Curriculum Framework]~~

~~AII.14~~ AII.5 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. ~~The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions. [Move to Curriculum Framework]~~

AII.13 ~~The student will solve practical problems, using systems of linear inequalities and linear programming, and describe the results both orally and in writing. A graphing calculator will be used to facilitate solutions to linear programming problems.~~

Functions

AII.8 ~~The student will recognize multiple representations of functions (absolute value, step, and exponential, functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators. [Moved to new SOL AII.6]~~

AII.6 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed.

AII.7 The student will investigate and analyze functions algebraically and graphically. Key concepts include:

- a) domain and range, including limited and discontinuous domains and ranges;
- b) zeros;
- c) x- and y-intercepts;
- d) intervals in which a function is increasing/decreasing;
- e) asymptotes;
- f) end behavior;
- g) inverse of a function; and
- h) composition of multiple functions.

AII.9 ~~The student will find the domain, [Moved to new SOL AII.7 a] range, [Moved to new SOL AII.7 a] zeros, [Moved to new SOL AII.7 b] and inverse of a function [Moved to new SOL AII.7 g]; the value of a function for a given element in its domain; [Moved to new SOL AII.7 f] and the composition of multiple functions. [Moved to new SOL AII.7 h] Functions will include exponential, logarithmic, [Move to Curriculum Framework] and those that have domains and ranges that are limited and/or discontinuous. [Moved to new SOL AII.7 a] The graphing calculator will be used as a tool to assist in investigation of functions. [Move to Curriculum Framework]~~

- ~~AII.15~~ ~~The student will recognize the general shape of polynomial, exponential, and logarithmic functions. [Moved to new SOL AII.6] The graphing calculator will be used as a tool to investigate the shape and behavior of these functions. [Move to Curriculum Framework]~~
- AII.8 The student will investigate and describe ~~through the use of graphs~~ the relationships between the solution of an equation, zero of a function, x-intercept of a graph, and factors of a polynomial expression.
- ~~AII.18~~ ~~The student will identify conic sections (circle, ellipse, parabola, and hyperbola) from his/her equations and graphs. Given the equations in (h, k) form, the student will sketch graphs of conic sections, using transformations.~~
- AII.9 The student will, given the coordinates of the center of a circle and a point on the circle, write the equation of the circle.

Statistics

- ~~AII.19~~ AII.10 The student will collect and analyze data, determine the equation of the curve of best fit, to make predictions, and solve practical real-world problems using mathematical models. Graphing calculators will be used to investigate scatterplots and to determine the equation for a curve of best fit. [Move to Curriculum Framework] Mathematical Mmodels will include linear, quadratic, polynomial, exponential, and logarithmic functions.
- ~~AII.20~~ AII.11 The student will identify, create, and solve ~~practical~~ real-world problems involving inverse variation and a combination of direct and inverse variations.
- AII.12 The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.
- AII.13 The student will compute and distinguish between permutations and combinations and use technology for applications.

Analytical Geometry

- ~~AII.10~~ ~~The student will investigate and describe through the use of graphs the relationships between the solution of an equation, zero of a function, x-intercept of a graph, and factors of a polynomial expression. [Move to Curriculum Framework in support of new SOL AII.7]~~

Systems of Equations and Inequalities

- AII.11 ~~The student will use matrix multiplication to solve practical problems. Graphing calculators or computer programs with matrix capabilities will be used to find the product.~~
- AII.12 ~~The student will represent problem situations with a system of linear equations and solve the system, using the inverse matrix method. Graphing calculators or computer programs with matrix capability will be used to perform computations.~~
- AII.14 ~~The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions. [Moved to new SOL AII.5]~~

Trigonometry

The standards below outline the content for a one-semester course in trigonometry. Students enrolled in trigonometry are assumed to have mastered those concepts outlined in the Algebra II standards. A thorough treatment of trigonometry is provided through the study of trigonometric definitions, applications, graphing, and solving trigonometric equations and inequalities. Emphasis should also be placed on using connections between right triangle ratios, trigonometric functions, and circular functions. In addition, applications and modeling should be included throughout the course of study. Emphasis should also be placed on oral and written communication concerning the language of mathematics, logic of procedure, and interpretation of results.

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of trigonometric functions and their inverses. They also provide a powerful tool for solving/verifying trigonometric equations and inequalities.

- T.1 The student will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of an angle in standard position, given a point, other than the origin, on the terminal side of the angle. ~~Circular function definitions will be connected with trigonometric function definitions.~~ Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.
- T.2 The student will, given the value of one trigonometric function, find the values of the other trigonometric functions. ~~Properties of the unit circle and, using the definitions and properties of circular the trigonometric functions, will be applied.~~
- T.3 The student will find without the aid of a ~~calculating utility~~ calculator the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting angle measures from radians to degrees and vice versa.
- T.4 The student will find with the aid of a calculator the value of any trigonometric function and inverse trigonometric function.
- T.5 The student will verify basic trigonometric identities and make substitutions, using the basic identities.

- T.6 The student, given one of the six trigonometric functions in standard form [~~e.g., $y = A \sin(Bx + C) + D$, where A , B , C , and D are real numbers~~], will
- state the domain and the range of the function;
 - determine the amplitude, period, phase shift, ~~and~~ vertical shift, ~~and~~ asymptotes;
 - sketch the graph of the function by using transformations for at least a ~~one~~ two-period interval; ~~and~~
 - The graphing calculator will be used to investigate the effect of changing A , B , C , and D the parameters in a trigonometric function on the graph of a trigonometric the function.
- T.7 The student will identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.
- T.8 The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities. ~~Graphing utilities will be used to solve equations, check for reasonableness of results, and verify algebraic solutions.~~ [Move to Curriculum Framework]
- T.9 The student will identify, create, and solve practical real-world problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

Algebra II and Trigonometry

The standards for this combined course in Algebra II and Trigonometry include all of the standards listed for Algebra II and Trigonometry. This course is designed for advanced students who are capable of a more rigorous course at an accelerated pace. The standards listed for this course provide the foundation for students to pursue a sequence of advanced mathematical studies from Mathematical Analysis to Advanced Placement Calculus.

Expressions and Operations

~~AII/T.1 The student will identify field properties, axioms of equality and inequality, and properties of order that are valid for the set of real numbers and its subsets, [Moved to new SOL A.4 b] complex numbers, [Moved to new SOL AII/T.3] and matrices.~~

~~AII/T.2~~ AII/T.1 The student will given rational, radical, or polynomial expressions,
 a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
 b) add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables, and expressions containing rational exponents;
 c) write radical expressions as expressions containing rational exponents and vice versa; and
 d) factor ~~completely~~ polynomials completely.

~~AII/T.2 The student will add, subtract, multiply, divide, and simplify rational expressions, [Moved to new SOL AII/T.1 a] including complex fractions. [Move to Curriculum Framework]~~

~~AII/T.3 The student will~~
 a) ~~add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables and expressions containing rational exponents; [Moved to new SOL AII/T.1 b] and~~
 b) ~~write radical expressions as expressions containing rational exponents and vice versa. [Moved to new SOL AII/T.1 c]~~

~~AII/T.5 The student will identify and factor completely polynomials [Moved to new SOL AII/T.1 d] representing the difference of squares, perfect square trinomials, the sum and difference of cubes, and general trinomials. [Move to Curriculum Framework]~~

~~AII/T.16~~ AII/T.2 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve practical real-world problems, including writing the first n terms, finding the n^{th} term, and evaluating summation formulas. Notation will include Σ and a_n .

~~AII/T.17 AII/T.3~~ The student will perform operations on complex numbers and express the results in simplest form. ~~Simplifying results will involve using patterns of the powers of i , and identify field properties that are valid for the complex numbers.~~

Equations and Inequalities

AII/T.4 The student will solve, algebraically and graphically,
 a) absolute value equations and inequalities;
 b) quadratic equations over the set of complex numbers;
 c) equations containing rational algebraic expressions; and
 d) equations containing radical expressions.

~~AII/T.4 The student will solve absolute value equations and inequalities. [Moved to new SOL AII/T.4 a] graphically and algebraically. [Moved to new SOL AII/T.4] Graphing calculators will be used as a primary method of solution and to verify algebraic solutions. [Move to Curriculum Framework]~~

~~AII/T.6 The student will select, justify, and apply a technique to solve a quadratic equation over the set of complex numbers. [Moved to new SOL AII/T.4 b] Graphing calculators will be used for solving and for confirming the algebraic solutions. [Move to Curriculum Framework]~~

~~AII/T.7 The student will solve equations containing rational expressions [Moved to new SOL AII/T.4 c] and equations containing radical expressions algebraically and graphically. [Moved to new SOL AII/T.4 d] Graphing calculators will be used for solving and for confirming the algebraic solutions. [Move to Curriculum Framework]~~

~~AII/T.14 AII/T.5~~ The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. ~~The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions. [Move to Curriculum Framework]~~

~~AII/T.13 The student will solve practical problems, using systems of linear inequalities and linear programming, and describe the results both orally and in writing. A graphing calculator will be used to facilitate solutions to linear programming problems.~~

Functions

~~AII/T.8 The student will recognize multiple representations of functions (absolute value, step, and exponential, functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators. [Moved to new SOL AII/T.6]~~

AII/T.6 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed.

- AII/T.7 The student will investigate and analyze functions algebraically and graphically. Key concepts include:
- a) domain and range, including limited and discontinuous domains and ranges;
 - b) zeros;
 - c) x- and y-intercepts;
 - d) intervals in which a function is increasing/decreasing;
 - e) asymptotes;
 - f) end behavior;
 - g) inverse of a function; and
 - h) composition of multiple functions.
- ~~AII/T.9~~ The student will find the domain, [Moved to new SOL AII/T.7 a] ~~range,~~ [Moved to new SOL AII/T.7 a] ~~zeros,~~ [Moved to new SOL AII/T.7 b] ~~and inverse of a function~~ [Moved to new SOL AII/T.7 g]; ~~the value of a function for a given element in its domain; and the composition of multiple functions.~~ [Moved to new SOL AII/T.7 h] Functions will include exponential, logarithmic, [Move to Curriculum Framework] and those that have domains and ranges that are limited and/or discontinuous. [Moved to new SOL AII/T.7 a] ~~The graphing calculator will be used as a tool to assist in investigation of functions.~~ [Move to Curriculum Framework]
- ~~AII/T.15~~ The student will recognize the general shape of polynomial, exponential, and logarithmic functions. [Moved to new SOL AII/T.6] ~~The graphing calculator will be used as a tool to investigate the shape and behavior of these functions.~~ [Move to Curriculum Framework]
- AII/T.8 The student will investigate and describe ~~through the use of graphs~~ the relationships between the solution of an equation, zero of a function, x-intercept of a graph, and factors of a polynomial expression.
- ~~AII/T.18~~ The student will identify conic sections (circle, ellipse, parabola, and hyperbola) from his/her equations and graphs. Given the equations in (h, k) form, the student will sketch graphs of conic sections, using transformations.
- AII/T.9 The student will, given the coordinates of the center of a circle and a point on the circle, write the equation of the circle.

Statistics

- ~~AII/T.19~~ AII/T.10 The student will collect and analyze data, determine the equation of the curve of best fit, ~~to make predictions,~~ and solve practical real-world problems using mathematical models. ~~Graphing calculators will be used to investigate scatterplots and to determine the equation for a curve of best fit.~~ [Move to Curriculum Framework] Mathematical Models will include linear, quadratic, polynomial, exponential, and logarithmic functions.

~~AII/T.20~~ AII/T.11 The student will identify, create, and solve ~~practical~~ real-world problems involving inverse variation and a combination of direct and inverse variations.

AII/T.12 The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.

AII/T.13 The student will compute and distinguish between permutations and combinations and use technology for applications

Analytical Geometry

~~AII/T.10~~ The student will investigate and describe through the use of graphs the relationships between the solution of an equation, zero of a function, x intercept of a graph, and factors of a polynomial expression. [Move to Curriculum Framework to support new SOL AII/T.7]

Systems of Equations and Inequalities

~~AII/T.11~~ The student will use matrix multiplication to solve practical problems. Graphing calculators or computer programs with matrix capabilities will be used to find the product.

~~AII/T.12~~ The student will represent problem situations with a system of linear equations and solve the system, using the inverse matrix method. Graphing calculators or computer programs with matrix capability will be used to perform computations.

~~AII/T.13~~ The student will solve practical problems, using systems of linear inequalities and linear programming, and describe the results both orally and in writing. A graphing calculator will be used to facilitate solutions to linear programming problems.

~~AII/T.14~~ The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions. [Moved to new SOL AII/T.5]

Trigonometry

~~AII/T.14~~ The student will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of an angle in standard position, given a point, other than the origin, on the terminal side of the angle. ~~Circular function definitions will be connected with trigonometric function definitions.~~ Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.

- AII/T.15 The student will, given the value of one trigonometric function, find the values of the other trigonometric functions. ~~Properties of the unit circle and using the definitions and properties of circular~~ the trigonometric functions will be applied.
- AII/T.16 The student will find without the aid of a ~~calculating utility~~ calculator the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting angle measures from radians to degrees and vice versa.
- AII/T.17 The student will find with the aid of a calculator the value of any trigonometric function and inverse trigonometric function.
- AII/T.18 The student will verify basic trigonometric identities and make substitutions, using the basic identities.
- AII/T.19 The student, given one of the six trigonometric functions in standard form ~~[e.g., $y = A \sin(Bx + C) + D$, where A , B , C , and D are real numbers]~~, will
- state the domain and the range of the function;
 - determine the amplitude, period, phase shift, ~~and~~ vertical shift, and asymptotes;
 - sketch the graph of the function by using transformations for at least a ~~one~~ two-period interval; and
 - ~~The graphing calculator will be used to investigate the effect of changing A , B , C , and D~~ the parameters in a trigonometric function on the graph of a trigonometric the function.
- AII/T.20 The student will identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.
- AII/T.21 The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities. ~~Graphing utilities will be used to solve equations, check for reasonableness of results, and verify algebraic solutions.~~ [Move to Curriculum Framework]
- AII/T.22 The student will identify, create, and solve practical problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

Computer Mathematics

This course is intended to provide students with experiences in using computer programming techniques and skills to solve problems that can be set up as mathematical models. Students enrolled in Computer Mathematics are assumed to have studied the concepts and skills in Algebra I and beginning geometry. Students who successfully complete the standards for this course may earn credit toward meeting the mathematics graduation requirement. It is recognized that many students will gain computer skills in other mathematics courses or in a separate curriculum outside of mathematics and prior to high school. In such cases, the standards indicated by an asterisk (*) should be included in the student's course of study and treated as a review.

Even though computer ideas should be introduced in the context of mathematical concepts, problem solving per se should be developed in the most general sense, making the techniques applicable by students in many other environments. Strategies include defining the problem; developing, refining, and implementing a plan; and testing and revising the solution. Programming, ranging from simple programs involving only a few lines to complex programs involving subprograms, should permeate the entire course and may include programming a graphing calculator or scripting a problem solution in a database or spreadsheet. Programming concepts, problem-solving strategies, and mathematical applications should be integrated throughout the course.

These standards identify fundamental principles and concepts in the field of computer science that will be used within the context of mathematical problem solving in a variety of applications. As students develop and refine skills in logic, organization, and precise expression, they will apply those skills to enhance learning in all disciplines.

- COM.1 The student will apply programming techniques and skills to solve ~~practical~~ real-world problems in mathematics arising from consumer, business, and other applications in mathematics. Problems will include opportunities for students to analyze data in charts, graphs, and tables and to use their knowledge of equations, formulas, and functions to solve these problems.
- *COM.2 The student will design, write, test, debug, and document a program. Programming documentation will include preconditions and postconditions of program segments, input/output specifications, the step-by-step plan, the test data, a sample run, and the program listing with appropriately placed comments.
- *COM.3 The student will write program specifications that define the constraints of a given problem. These specifications will include descriptions of pre-conditions, post-conditions, the desired output, analysis of the available input, and an indication as to whether or not the problem is solvable under the given conditions.

- *COM.4 The student will design a step-by-step plan (algorithm) to solve a given problem. The plan will be in the form of a program flowchart, pseudo code, hierarchy chart, and/or data-flow diagram.
- *COM.5 The student will divide a given problem into manageable sections (modules) by task and implement the solution. The modules will include an appropriate user-defined function, subroutines, and procedures. Enrichment topics might include user-defined libraries (units) and object-oriented programming.
- *COM.6 The student will design and implement the input phase of a program, which will include designing screen layout and getting information into the program by way of user interaction, data statements, and/or file input. The input phase also will include methods of filtering out invalid data (error trapping).
- *COM.7 The student will design and implement the output phase of a computer program, which will include designing output layout, accessing a variety of output devices, using output statements, and labeling results.
- COM.8 The student will design and implement computer graphics, which will include topics appropriate for the available programming environment as well as student background. Students will use graphics as an end in itself, as an enhancement to other output, and as a vehicle for reinforcing programming techniques.
- COM.9 The student will define simple variable data types that include integer, real (fixed and scientific notation), character, string, and Boolean.
- COM.10 The student will use appropriate variable data types, including integer, real (fixed and scientific notation), character, string, and Boolean. This will also include variables representing structured data types.
- COM.11 The student will describe the way the computer stores, accesses, and processes variables, including the following topics: the use of variables versus constants, variables addresses, pointers, parameter passing, scope of variables, and local versus global variables.
- *COM.12 The student will translate a mathematical expression into a computer statement, which involves writing assignment statements and using the order of operations.
- COM.13 The student will select and implement built-in (library) functions in processing data.
- COM.14 The student will implement conditional statements that include "if/then" statements, "if/then/else" statements, case statements, and Boolean logic.
- COM.15 The student will implement loops, including iterative loops. Other topics will include single entry point, single exit point, preconditions, and postconditions.

- COM.16 The student will select and implement appropriate data structures, including arrays (one-dimensional and/or multidimensional), files, and records. Implementation will include creating the data structure, putting information into the structure, and retrieving information from the structure.
- *COM.17 The student will implement pre-existing algorithms, including sort routines, search routines, and simple animation routines.
- COM.18 The student will test a program, using an appropriate set of data. The set of test data should be appropriate and complete for the type of program being tested.
- COM.19 The student will debug a program, using appropriate techniques (e.g., appropriately placed controlled breaks, the printing of intermediate results, and other debugging tools available in the programming environment), and identify the difference between syntax errors and logic errors.
- COM.20 The student will design, write, test, debug, and document a complete structured program that requires the synthesis of many of the concepts contained in previous standards.

Probability and Statistics

The following standards outline the content of a one-year course in Probability and Statistics. If a one-semester course is desired, the standards with an asterisk (*) would apply. Students enrolled in this course are assumed to have mastered the concepts identified in the Standards of Learning for Algebra II. The purpose of the course is to present basic concepts and techniques for collecting and analyzing data, drawing conclusions, and making predictions.

A graphing calculator is essential for every student taking the Probability and Statistics course and is required for the Advanced Placement Statistics Examination. The calculator may not fully substitute for a computer, however. In the absence of a computer for student use, teachers may provide students with examples of computer output generated by a statistical software package.

- *PS.1 The student will analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.

- *PS.2 The student will analyze numerical characteristics of univariate data sets to describe patterns and departure from patterns, using mean, median, mode, variance, standard deviation, interquartile range, range, and outliers. ~~Appropriate technology will be used to calculate statistics.~~ [Move to Curriculum Framework]

- *PS.3 The student will compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features. ~~Appropriate technology will be used to generate graphical displays.~~[Move to Curriculum Framework]

- *PS.4 The student will analyze scatterplots to identify and describe the relationship between two variables, using shape; strength of relationship; clusters; positive, negative, or no association; outliers; and influential points. ~~Appropriate technology will be used to generate scatterplots and identify outliers and influential points.~~ [Move to Curriculum Framework]

- PS.5 The student will ~~find and interpret linear correlation, use the method of least squares regression to model the linear relationship between two variables, and use the residual plots to assess linearity.~~ Appropriate technology will be used to compute correlation coefficients and residual plots. compare and contrast methods of determining regression models

- PS.6 The student will make logarithmic and power transformations to achieve linearity. ~~Appropriate technology will be used.~~ [Move to Curriculum Framework]

- PS.7 The student, using two-way tables, will analyze categorical data to describe patterns and departure from patterns and to find marginal frequency and relative frequencies, including conditional frequencies.

- *PS.8 The student will describe the methods of data collection in a census, sample survey, experiment, and observational study and identify an appropriate method of solution for a given problem setting.
- *PS.9 The student will plan and conduct a survey. The plan will address sampling techniques (e.g., simple random and stratified) and methods to reduce bias.
- PS.10 The student will plan and conduct an experiment. The plan will address control, randomization, and measurement of experimental error.
- ~~*PS.11 The student will compute and distinguish between permutations and combinations and use technology for applications. [Moved to new SOL AII.13 and new SOL AII/T.13]~~
- *PS.11 The student will identify and describe two or more events as complementary, dependent, independent, and/or mutually exclusive.
- *PS.12 The student will find probabilities (relative frequency and theoretical), including conditional probabilities for events that are either dependent or independent, by applying the “law of large numbers” concept, the addition rule, and the multiplication rule.
- *PS.13 The student will develop, interpret, and apply the binomial probability distribution for discrete random variables, including computing the mean and standard deviation for the binomial variable.
- PS.14 The student will simulate probability distributions, including binomial and geometric.
- PS.15 The student will identify random variables as independent or dependent and find the mean and standard deviations for sums and differences of independent random variables.
- ~~*PS.17 The student will identify properties of a normal distribution and apply the normal distribution to determine probabilities, using a table or graphing calculator. [Moved to new SOL AII.12 and new SOL AII/T.12]~~
- *PS.16 The student, given data from a large sample, will find and interpret point estimates and confidence intervals for parameters. The parameters will include proportion and mean, difference between two proportions, and difference between two means (independent and paired).
- PS.17 The student will apply and interpret the logic of a hypothesis-testing procedure. Tests will include large sample test for proportion, mean, difference between two proportions, and difference between two means (independent and paired) and Chi-squared test for goodness of fit, homogeneity of proportions, and independence.

- PS.18 The student will identify the meaning of sampling distribution with reference to random variable, sampling statistic, and parameter and explain the Central Limit Theorem. This will include sampling distribution of a sample proportion, a sample mean, a difference between two sample proportions, and a difference between two sample means.
- PS.19 The student will identify properties of a t-distribution and apply t-distributions to single-sample and two-sample (independent and matched pairs) t-procedures, using tables or graphing calculators.

Discrete Mathematics

The following standards outline the content of a one-year course in Discrete Mathematics. If a one-semester course is desired, the standards with an asterisk (*) would apply. Students enrolled in Discrete Mathematics are assumed to have mastered the concepts outlined in the Standards of Learning for Algebra II.

Discrete mathematics may be described as the study of mathematical properties of sets and systems that have a countable (discrete) number of elements. With the advent of modern technology, discrete (discontinuous) models have become as important as continuous models. In this course, the main focus is problem solving in a discrete setting. Techniques that are not considered in the current traditional courses of algebra, geometry, and calculus will be utilized. As students solve problems, they will analyze and determine whether or not a solution exists (existence problems), investigate how many solutions exist (counting problems), and focus on finding the best solution (optimization problems). Connections will be made to other disciplines. The importance of discrete mathematics has been influenced by computers. ~~Modern~~ Technology (graphing calculators and/or computers) will be an integral component of this course.

- *DM.1 The student will model problems, using vertex-edge graphs. The concepts of valence, connectedness, paths, planarity, and directed graphs will be investigated. Adjacency matrices and matrix operations will be used to solve problems (e.g., food chains, number of paths).
- *DM.2 The student will solve problems through investigation and application of circuits, cycles, Euler Paths, Euler Circuits, Hamilton Paths, and Hamilton Circuits. Optimal solutions will be sought using existing algorithms and student-created algorithms.
- *DM.3 The student will apply graphs to conflict-resolution problems, such as map coloring, scheduling, matching, and optimization. Graph coloring and chromatic number will be used.
- *DM.4 The student will apply algorithms, such as Kruskal's, Prim's, or Dijkstra's, relating to trees, networks, and paths. Appropriate technology will be used to determine the number of possible solutions and generate solutions when a feasible number exists.
- *DM.5 The student will use algorithms to schedule tasks in order to determine a minimum project time. The algorithms will include critical path analysis, the list-processing algorithm, and student-created algorithms.
- *DM.6 The student will solve linear programming problems. Appropriate technology will be used to facilitate the use of matrices, graphing techniques, and the Simplex method of determining solutions.

- *DM.7 The student will analyze and describe the issue of fair division (e.g., cake cutting, estate division). Algorithms for continuous and discrete cases will be applied.
- DM.8 The student will investigate and describe weighted voting and the results of various election methods. These may include approval and preference voting as well as plurality, majority, run-off, sequential run-off, Borda count, and Condorcet winners.
- DM.9 The student will identify apportionment inconsistencies that apply to issues such as salary caps in sports and allocation of representatives to Congress. Historical and current methods will be compared.
- DM.10 The student will use the recursive process and difference equations with the aid of appropriate technology to generate
 - a) compound interest;
 - b) sequences and series;
 - c) fractals;
 - d) population growth models; and
 - e) the Fibonacci sequence.
- DM.11 The student will describe and apply sorting algorithms and coding algorithms used in storing, processing, and communicating information. These will include
 - a) bubble sort, merge sort, and network sort; and
 - b) ISBN, UPC, Zip, and banking codes.
- DM.12 The student will select, justify, and apply an appropriate technique to solve a logic problem. Techniques will include Venn diagrams, truth tables, and matrices.
- DM.13 The student will apply the formulas of combinatorics in the areas of
 - a) the Fundamental (Basic) Counting Principle;
 - b) knapsack and bin-packing problems;
 - c) permutations and combinations; and
 - d) the pigeonhole principle.

Mathematical Analysis

The standards below outline the content for a one-year course in Mathematical Analysis. Students enrolled in Mathematical Analysis are assumed to have mastered Algebra II concepts and have some exposure to trigonometry. Mathematical Analysis develops students' understanding of algebraic and transcendental functions, parametric and polar equations, sequences and series, and vectors. The content of this course serves as appropriate preparation for a calculus course.

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of functions and their inverses. They also provide a powerful tool for solving and verifying equations and inequalities.

- MA.1 The student will investigate and identify the characteristics of polynomial and rational functions and use these to sketch the graphs of the functions. This will include determining zeros, upper and lower bounds, y -intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. ~~Graphing utilities will be used to investigate and verify these characteristics.~~ [Move to Curriculum Framework]

- MA.2 The student will ~~find~~ apply compositions of functions and inverses of functions to real-world situations. Analytical methods and graphing utilities will be used to investigate and verify the domain and range of resulting functions.

- MA.3 The student will investigate and describe the continuity of functions using graphs and algebraic methods. ~~The functions will include piecewise and step functions.~~ [Move to Curriculum Framework]

- MA.4 The student will expand binomials having positive integral exponents through the use of the Binomial Theorem, the formula for combinations, and Pascal's Triangle.

- MA.5 The student will find the sum (sigma notation included) of finite and infinite convergent series that will lead to an intuitive approach to a limit.

- MA.6 The student will use mathematical induction to prove formulas/statements.

- MA.7 The student will find the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. A graphing utility will be used to verify intuitive reasoning, algebraic methods, and numerical substitution.

- MA.8 The student will investigate and identify the characteristics of conic section equations in (h, k) and standard forms. The techniques of translations in the coordinate plane will be used to graph conic sections.

- MA.9 The student will investigate and identify the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and ~~practical~~ real-world problems. This will include the role of e , natural and common logarithms, laws of exponents and logarithms, and the solution of logarithmic and exponential equations. ~~Graphing utilities will be used to investigate and verify the graphs and solutions.~~ [Move to Curriculum Framework]
- MA.10 The student will investigate and identify the characteristics of the graphs of polar equations, using graphing utilities. This will include classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations.
- MA.11 The student will perform operations with vectors in the coordinate plane and solve ~~practical~~ real-world problems using vectors. This will include the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.
- MA.12 The student will use parametric equations to model and solve application problems. ~~Graphing utilities will be used to develop an understanding of the graph of parametric equations.~~ [Move to Curriculum Framework]
- MA.13 The student will identify, create, and solve ~~practical~~ real-world problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

Advanced Placement Calculus

[The College Board publishes the curricula for all Advanced Placement courses and updates these curricula biennially. Four comments from the public suggested that the course code for Advanced Placement Calculus be maintained but the Standards of Learning be deleted since teachers use the materials published by The College Board to guide instruction. The deletion of these Standards of Learning would be consistent with AP Statistics and AP Computer Science in the list of mathematics courses approved by the Board of Education to be used for high school graduation in Virginia.]

- APC.1 — ~~The student will define and apply the properties of elementary functions, including algebraic, trigonometric, exponential, and composite functions and their inverses, and graph these functions, using a graphing calculator. Properties of functions will include domains, ranges, combinations, odd, even, periodicity, symmetry, asymptotes, zeros, upper and lower bounds, and intervals where the function is increasing or decreasing.~~
- APC.2 — ~~The student will define and apply the properties of limits of functions. Limits will be evaluated graphically and algebraically. This will include~~
 a) ~~limits of a constant;~~
 b) ~~limits of a sum, product, and quotient;~~
 c) ~~one-sided limits; and~~
 d) ~~limits at infinity, infinite limits, and non-existent limits.*~~
 *AP Calculus BC will include l'Hopital's Rule, which will be used to find the limit of functions whose limits yield the indeterminate forms: $0/0$ and ∞/∞ .
- APC.3 — ~~The student will use limits to define continuity and determine where a function is continuous or discontinuous. This will include~~
 a) ~~continuity in terms of limits;~~
 b) ~~continuity at a point and over a closed interval;~~
 c) ~~application of the Intermediate Value Theorem and the Extreme Value Theorem; and~~
 d) ~~geometric understanding and interpretation of continuity and discontinuity.~~
- APC.4 — ~~The student will investigate asymptotic and unbounded behavior in functions. This will include~~
 a) ~~describing and understanding asymptotes in terms of graphical behavior and limits involving infinity; and~~
 b) ~~comparing relative magnitudes of functions and their rates of change.~~

- APC.5 — The student will investigate derivatives presented in graphic, numerical, and analytic contexts and the relationship between continuity and differentiability. The derivative will be defined as the limit of the difference quotient and interpreted as an instantaneous rate of change.
- APC.6 — The student will investigate the derivative at a point on a curve. This will include
- a) ~~finding the slope of a curve at a point, including points at which the tangent is vertical and points at which there are no tangents;~~
 - b) ~~using local linear approximation to find the slope of a tangent line to a curve at the point;~~
 - c) defining instantaneous rate of change as the limit of average rate of change; and
 - d) approximating rate of change from graphs and tables of values.
- APC.7 — The student will analyze the derivative of a function as a function in itself. This will include
- a) ~~comparing corresponding characteristics of the graphs of f , f' , and f'' ;~~
 - b) ~~defining the relationship between the increasing and decreasing behavior of f and the sign of f' ;~~
 - c) ~~translating verbal descriptions into equations involving derivatives and vice versa;~~
 - d) ~~analyzing the geometric consequences of the Mean Value Theorem;~~
 - e) ~~defining the relationship between the concavity of f and the sign of f'' ;~~
and
 - f) ~~identifying points of inflection as places where concavity changes and finding points of inflection.~~
- APC.8 — The student will apply the derivative to solve problems. This will include
- a) ~~analysis of curves and the ideas of concavity and monotonicity;~~
 - b) ~~optimization involving global and local extrema;~~
 - c) ~~modeling of rates of change and related rates;~~
 - d) ~~use of implicit differentiation to find the derivative of an inverse function;~~
 - e) ~~interpretation of the derivative as a rate of change in applied contexts, including velocity, speed, and acceleration; and~~
 - f) ~~differentiation of nonlogarithmic functions, using the technique of logarithmic differentiation.*~~
- * AP Calculus BC will also apply the derivative to solve problems. This will include
- a) ~~analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration vectors;~~
 - b) ~~numerical solution of differential equations, using Euler's method;~~
 - c) ~~l'Hopital's Rule to test the convergence of improper integrals and series; and~~
 - d) ~~geometric interpretation of differential equations via slope fields and the~~

relationship between slope fields and the solution curves for the differential equations.

- APC.9 The student will apply formulas to find derivatives. This will include
- a) ~~derivatives of algebraic, trigonometric, exponential, logarithmic, and inverse trigonometric functions;~~
 - b) ~~derivations of sums, products, quotients, inverses, and composites (chain rule) of elementary functions;~~
 - c) ~~derivatives of implicitly defined functions; and~~
 - d) ~~higher order derivatives of algebraic, trigonometric, exponential, and logarithmic, functions.*~~

* AP Calculus BC will also include finding derivatives of parametric, polar, and vector functions.

- APC.10 The student will use Riemann sums and the Trapezoidal Rule to approximate definite integrals of functions represented algebraically, graphically, and by a table of values and will interpret the definite integral as the accumulated rate of change of a quantity over an interval interpreted as the change of the quantity over the interval

$$\int_a^b f'(x) dx = f(b) - f(a).$$

Riemann sums will use left, right, and midpoint evaluation points over equal subdivisions.

- APC.11 The student will find antiderivatives directly from derivatives of basic functions and by substitution of variables (including change of limits for definite integrals).*

* AP Calculus BC will also include finding antiderivatives by parts and simple partial fractions (nonrepeating linear factors only), and finding improper integrals as limits of definite integrals.

* AP Calculus BC will also solve logistic differential equations and use them in modeling.

- APC.12 The student will identify the properties of the definite integral. This will include additivity and linearity, the definite integral as an area, and the definite integral as a limit of a sum as well as the fundamental theorem:

$$\frac{d}{dx} \int_a^x f(t) d(t) = f(x).$$

- APC.13 The student will use the Fundamental Theorem of Calculus to evaluate definite integrals, represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

- APC.14 The student will find specific antiderivatives, using initial conditions (including applications to motion along a line). Separable differential equations will be

solved and used in modeling (in particular, the equation $y' = ky$ and exponential growth).

- APC.15 — The student will use integration techniques and appropriate integrals to model physical, biological, and economic situations. The emphasis will be on using the integral of a rate of change to give accumulated change or on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. Specific applications will include
- a) — the area of a region;
 - b) — the volume of a solid with known cross-section;
 - c) — the average value of a function; and
 - d) — the distance traveled by a particle along a line.*

* AP Calculus BC will include finding the area of a region (including a region bounded by polar curves) and finding the length of a curve (including a curve given in parametric form).

- APC.16 — The student will define a series and test for convergence of a series in terms of the limit of the sequence of partial sums. This will include
- a) — geometric series with applications;
 - b) — harmonic series;
 - c) — alternating series with error bound;
 - d) — terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p -series; and
 - e) — ratio test for convergence and divergence.*

* For those students who are enrolled in AP Calculus BC.

- APC.17 — The student will define, restate, and apply Taylor series. This will include
- a) — Taylor polynomial approximations with graphical demonstration of convergence;
 - b) — Maclaurin series and the general Taylor series centered at $x = a$;
 - c) — Maclaurin series for the functions e^x , $\sin x$, $\cos x$, and $1/(1-x)$;
 - d) — formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series;
 - e) — functions defined by power series;
 - f) — radius and interval of convergence of power series; and
 - g) — Lagrange error bound of a Taylor polynomial.*

* For those students who are enrolled in AP Calculus BC.